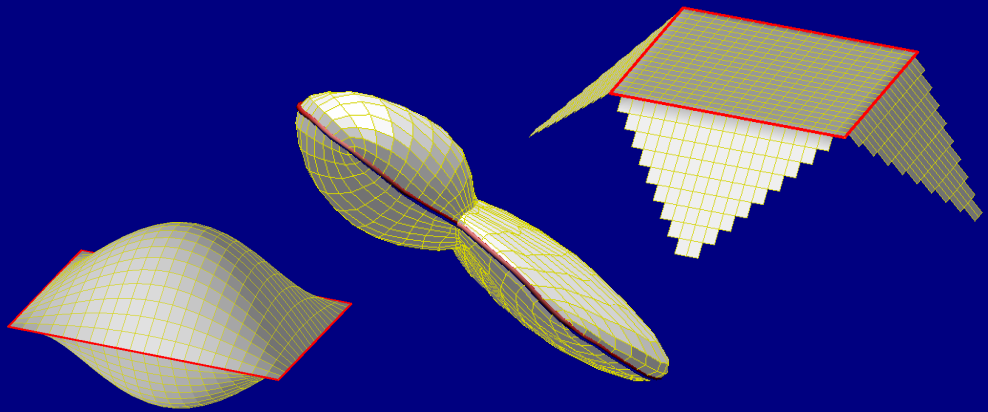
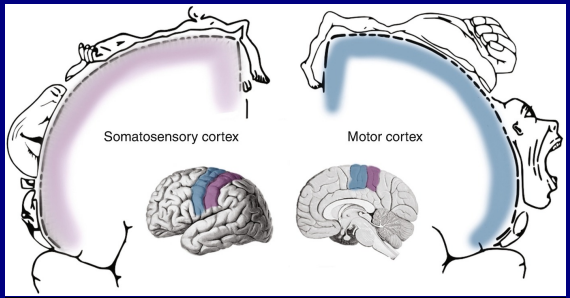
An unambiguous global map projection for the Kuiper belt object Arrokoth by fitting a Quincuncial Adaptive Closed Kohonen (QuACK) map

Björn Grieger (Aurora Technology B. V. for the European Space Agency, ESAC, Spain)



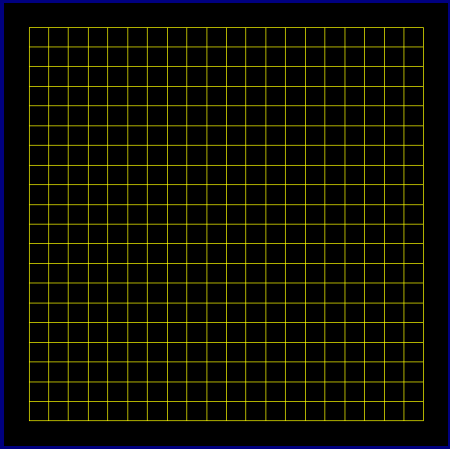
Sensory and motor cortex



Male and female sensory humunculi

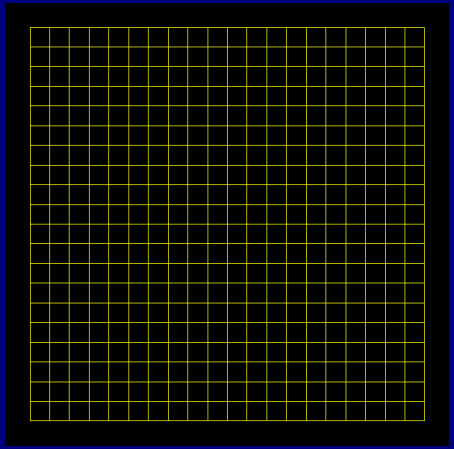
First female humunculus from Write and Foerder (2020).

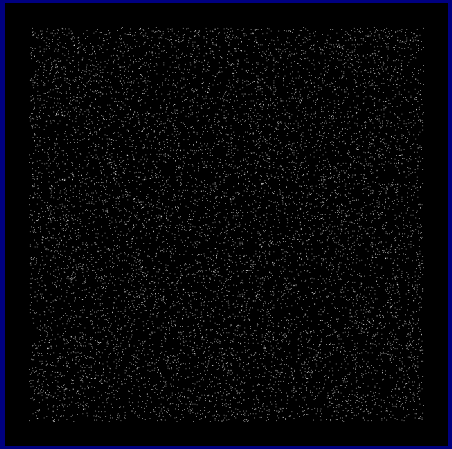
Fitting a map to a square



Low resolution (21×21) toy map

Fitting a map to a square

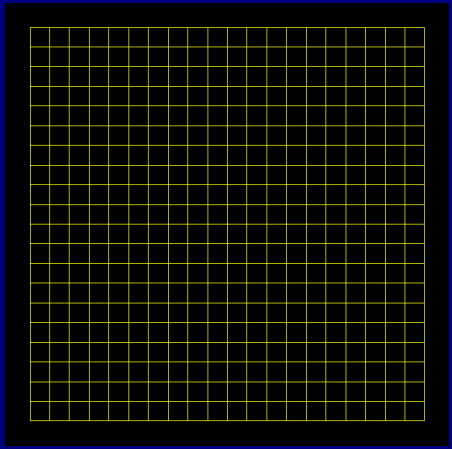




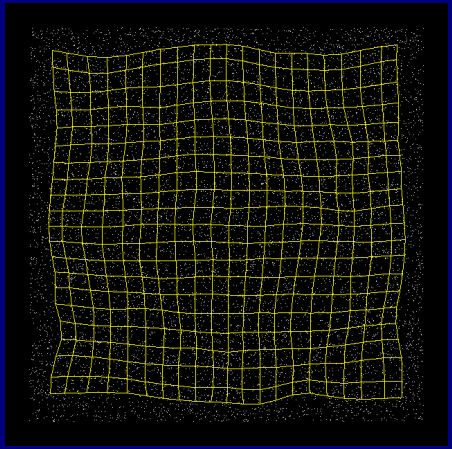
Low resolution (21×21) toy map

10 000 random points in a square

Fitting a map to a square

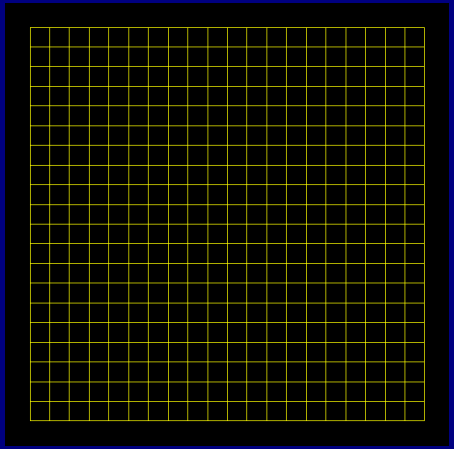


Low resolution (21×21) toy map

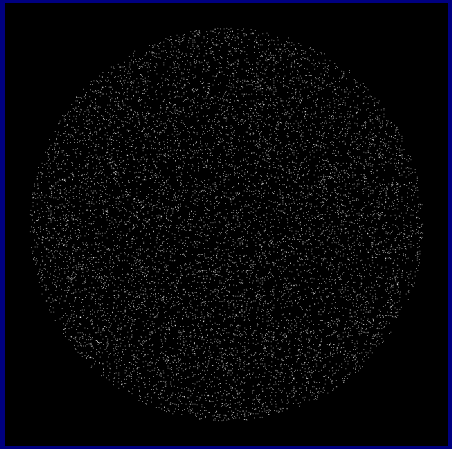


Self-organized Kohonen map

Fitting a map to a circle

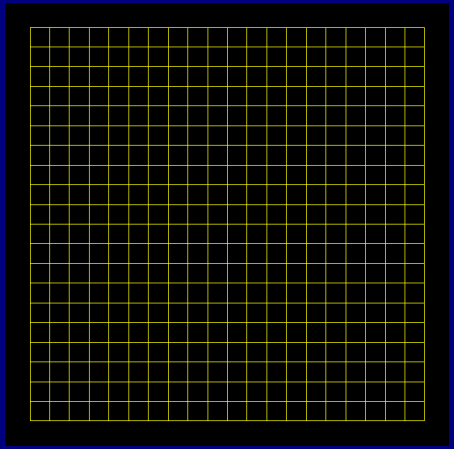


Low resolution (21×21) toy map

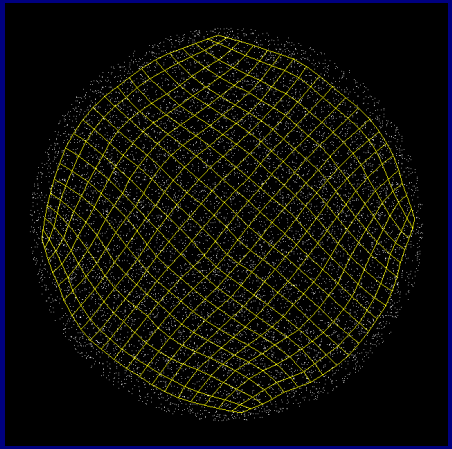


10 000 random points in a circle

Fitting a map to a circle

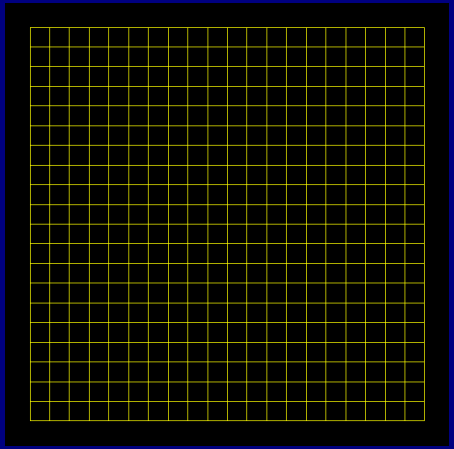


Low resolution (21×21) toy map

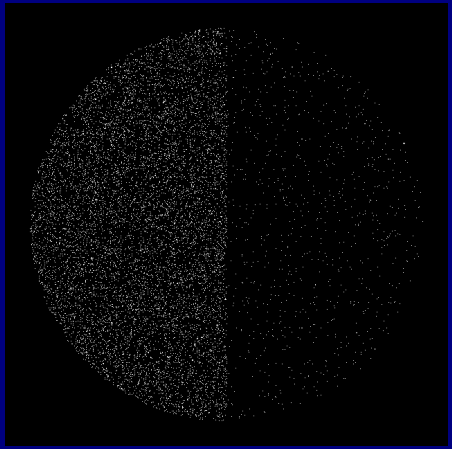


"Squaring the circle" 😀

Fitting a map to a circle with varying data density

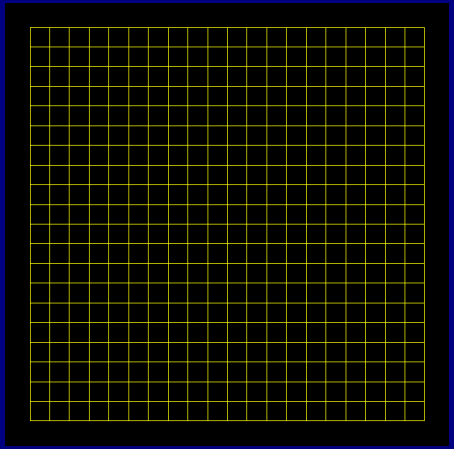


Low resolution (21×21) toy map

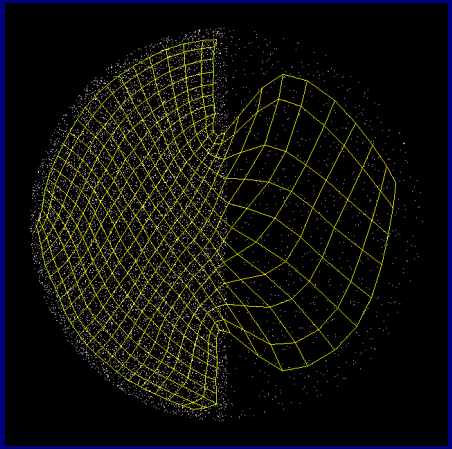


Factor 10 different density

Fitting a map to a circle with varying data density

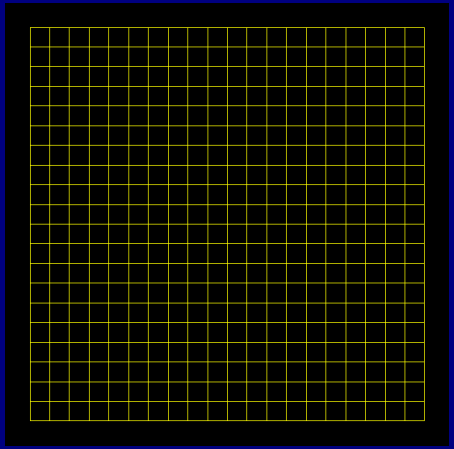


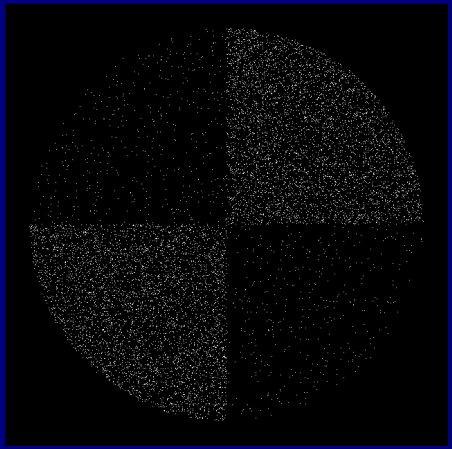
Low resolution (21×21) toy map



Self-organized Kohonen map

Fitting a map to a circle with varying data density

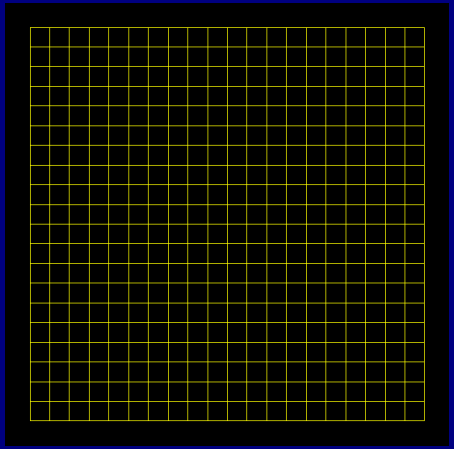




Low resolution (21×21) toy map

Factor 10 different density

Fitting a map to a circle with varying data density



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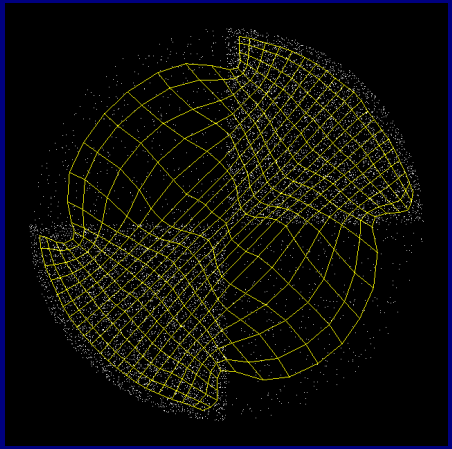
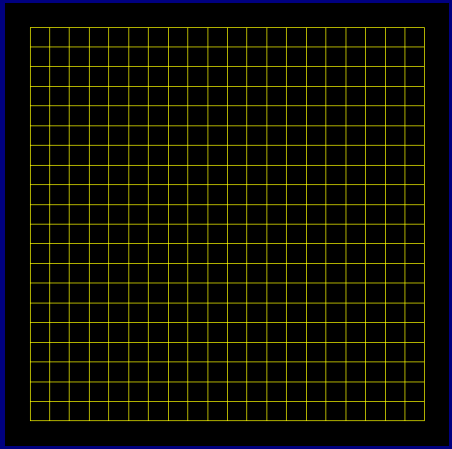
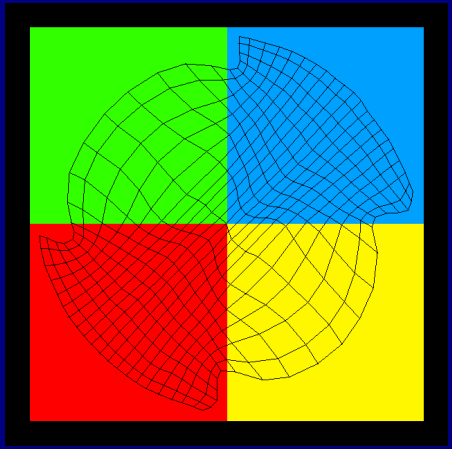


Self-organized Kohonen map

Mapping features

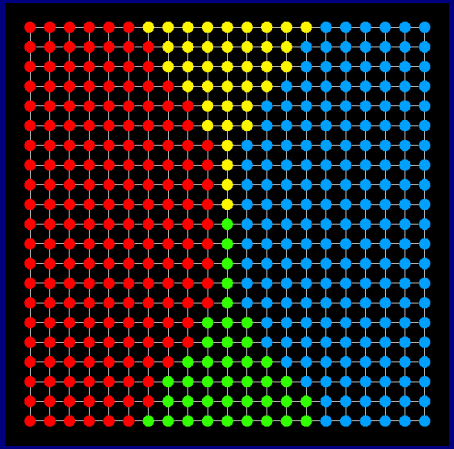


Low resolution (21×21) toy map

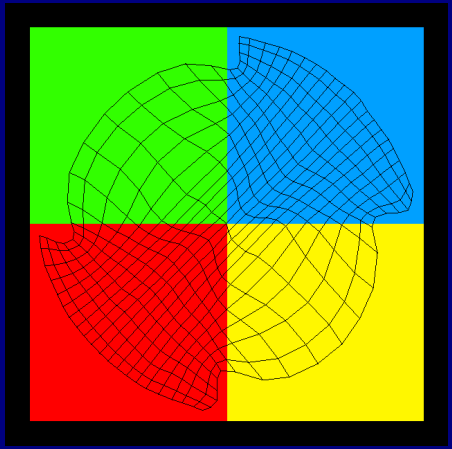


Regions have "features" (represented by color).

Mapping features

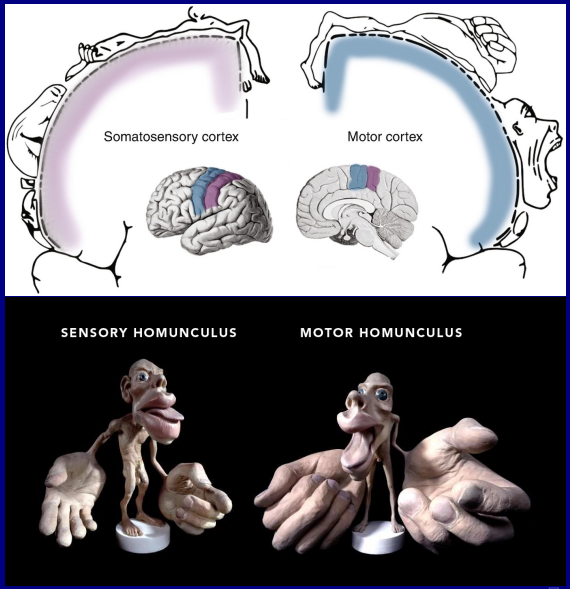


Topologically correct feature map

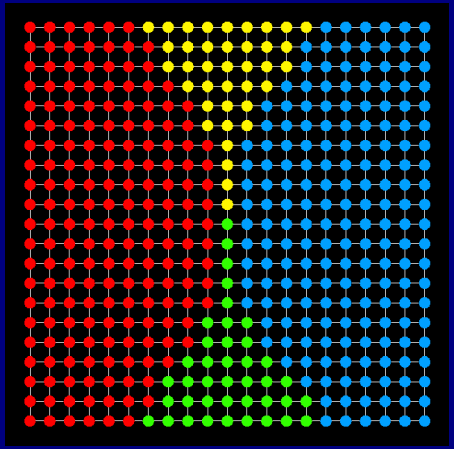


Regions have “features” (represented by color).

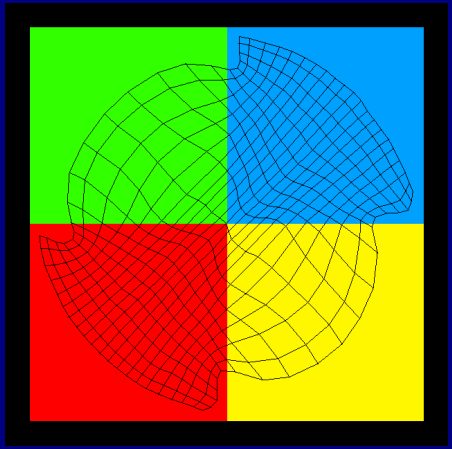
Sensory and motor cortex



Mapping features

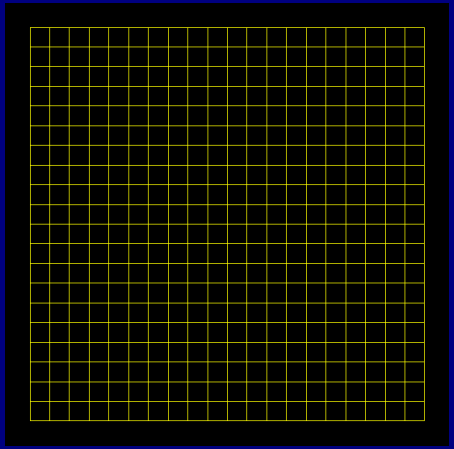


Topologically correct feature map



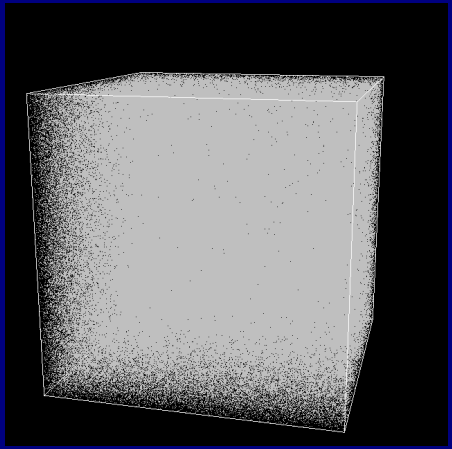
Regions have “features” (represented by color).

Fitting a 2D map to a 3D volume



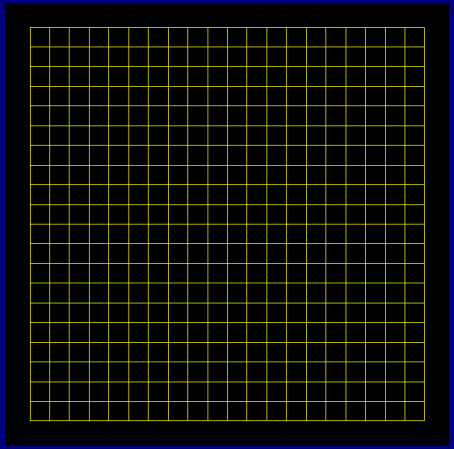
Low resolution (21×21) toy map

$\longrightarrow$

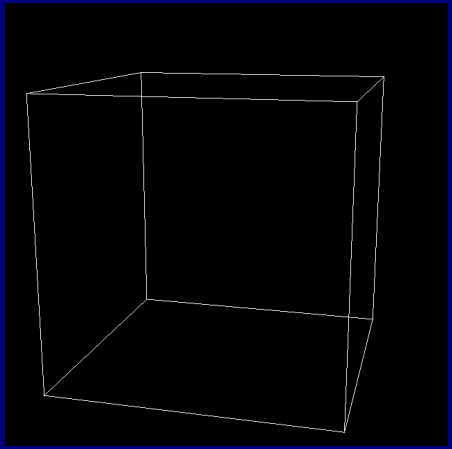


1 000 000 random points in a cube

Fitting a 2D map to a 3D volume

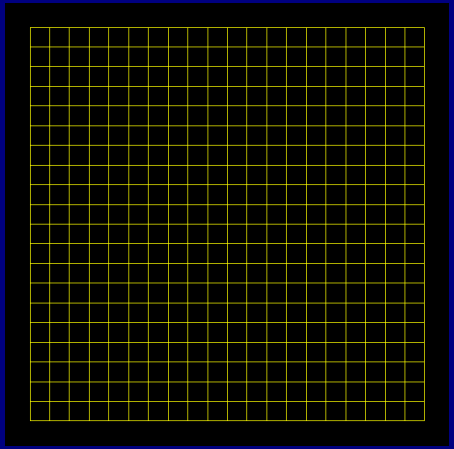


Low resolution (21×21) toy map

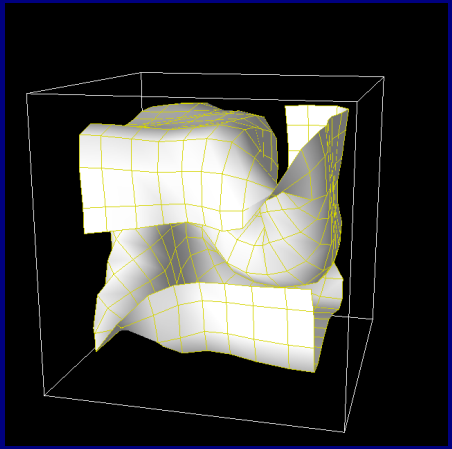


What will the map do?

Fitting a 2D map to a 3D volume

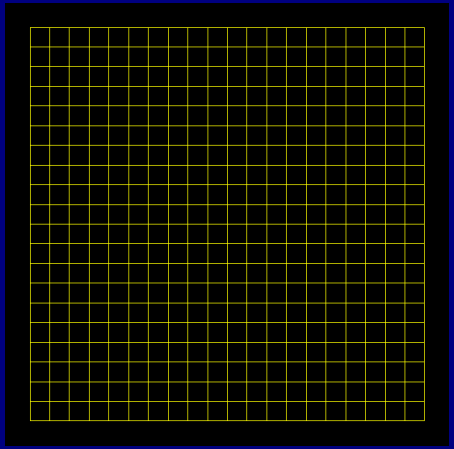


Low resolution (21×21) toy map



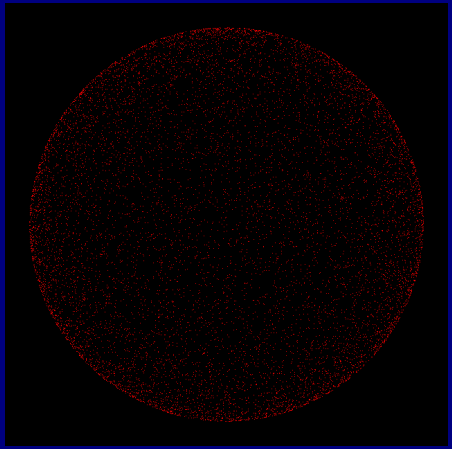
It tries its best!

Fitting a map to the surface of a sphere



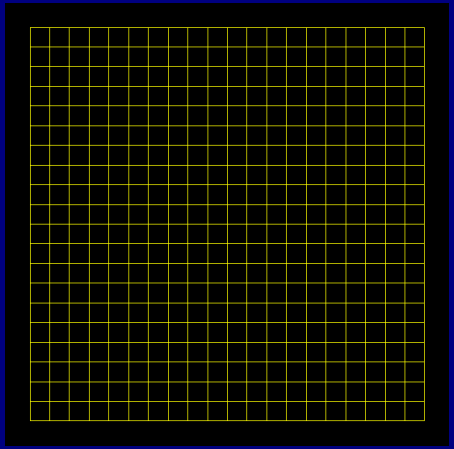
Low resolution (21×21) toy map

$\longrightarrow$

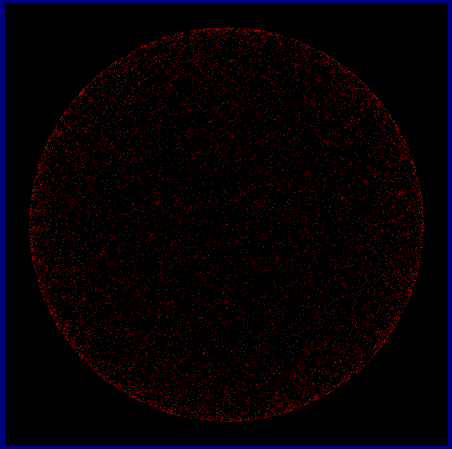


10 000 random points on the surface of a sphere

Iteration 0

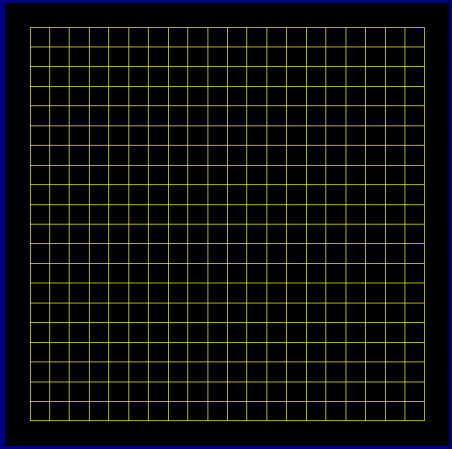


Low resolution (21×21) toy map



Initial small random numbers

Iteration 0 — zoom



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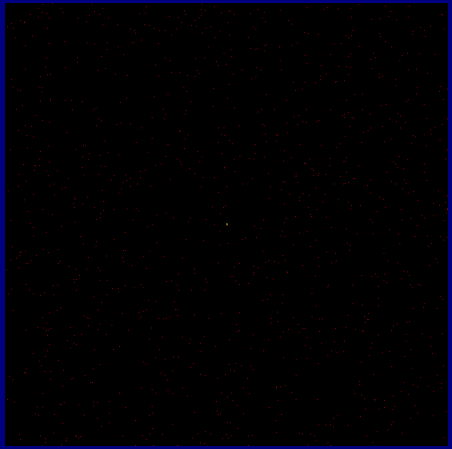
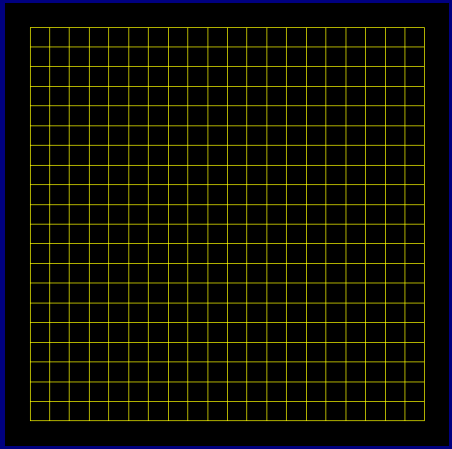


Initial small random numbers

Iteration 0 — zoom



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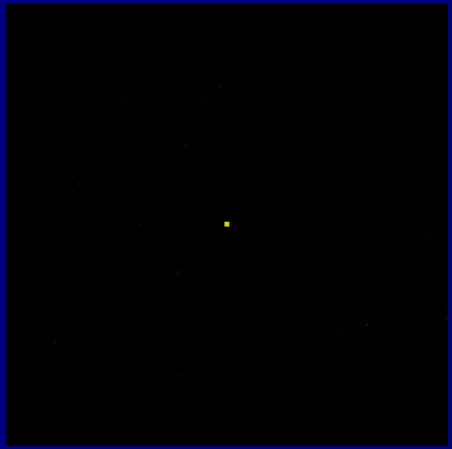
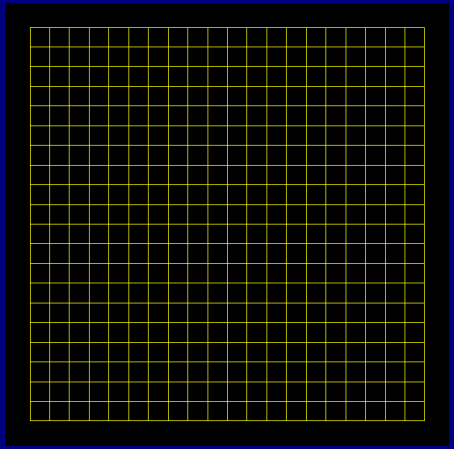


Initial small random numbers

Iteration 0 — zoom



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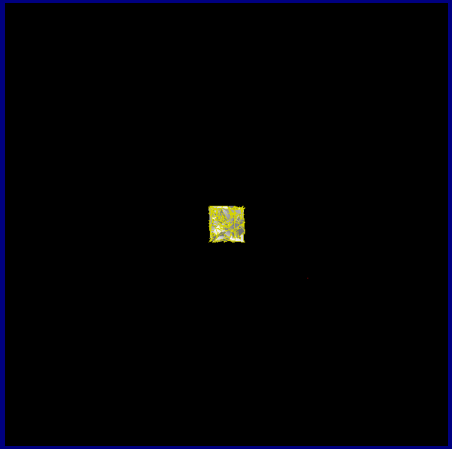
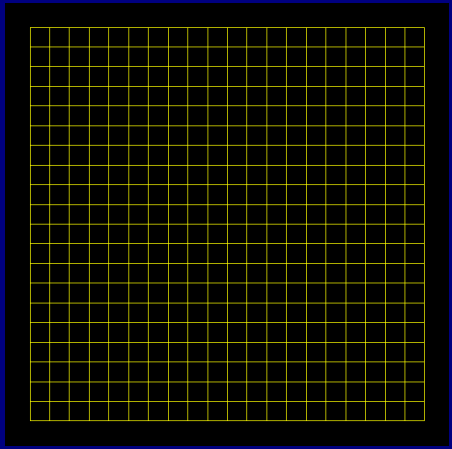


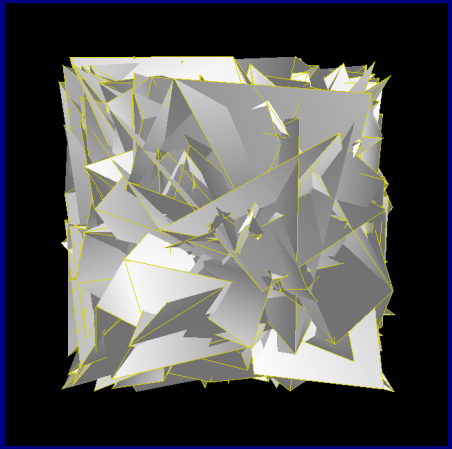
Initial small random numbers

Iteration 0 — zoom



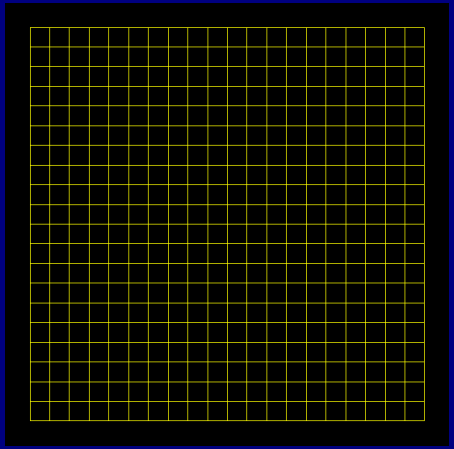
Low resolution (21×21) toy map

$\longrightarrow$



Initial small random numbers

Iteration 0



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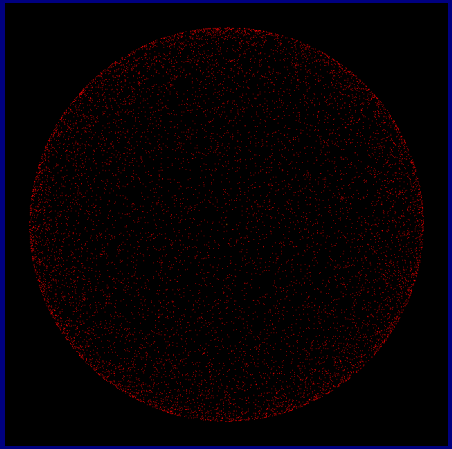
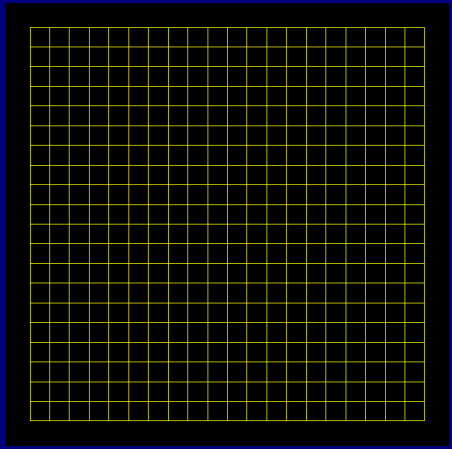


Initial small random numbers

Iteration 1



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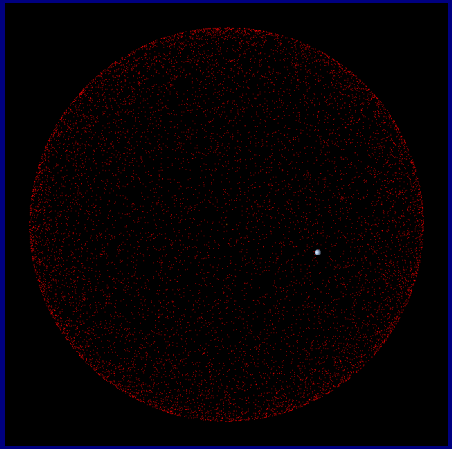
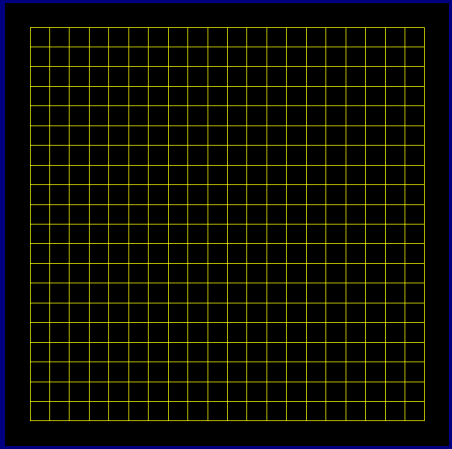


Pick random data point

Iteration 1



Low resolution (21×21) toy map

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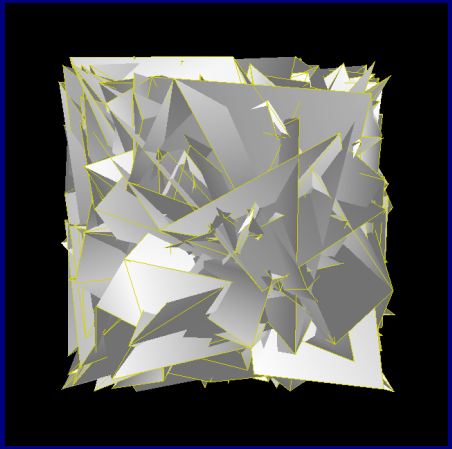
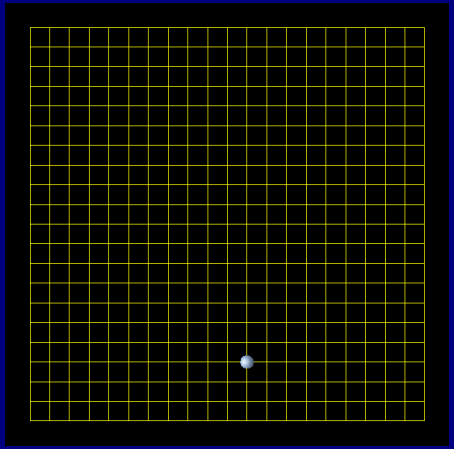


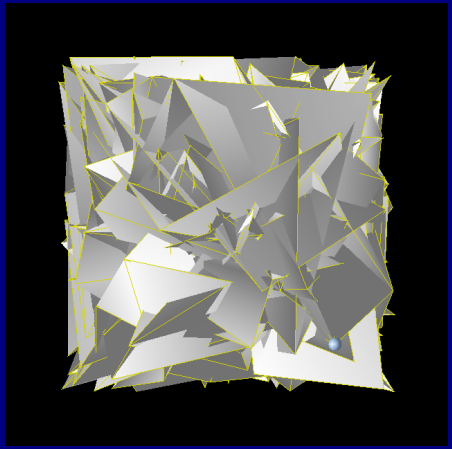
Find closest point on map

Iteration 1



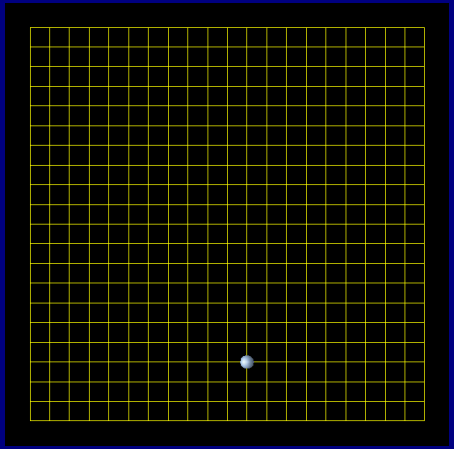
Low resolution (21×21) toy map

$\longrightarrow$



Find closest point on map

Iteration 1



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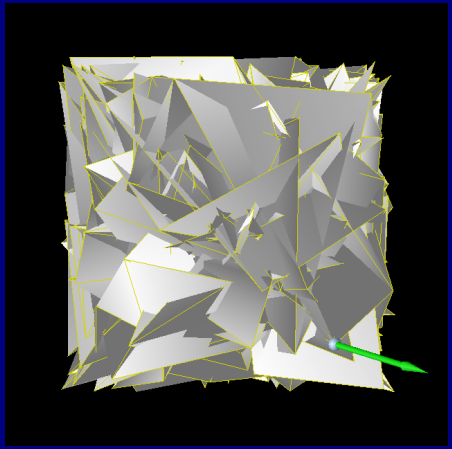
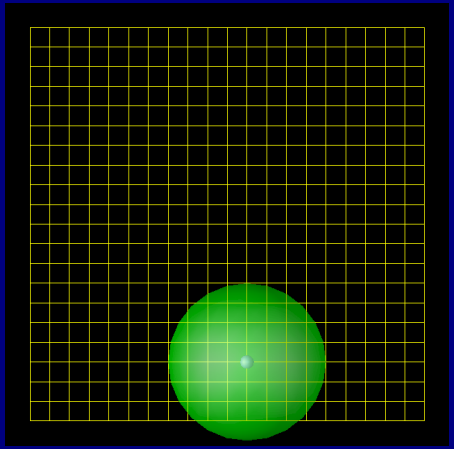
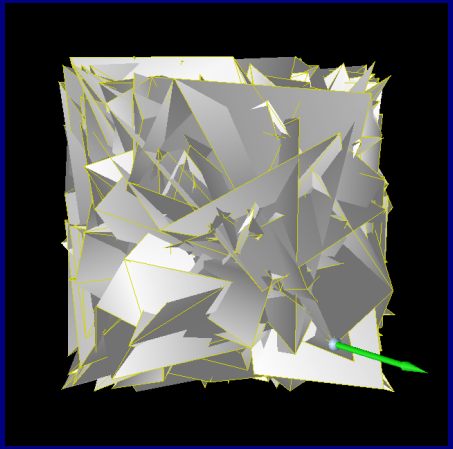


Tear it towards the data point

Iteration 1

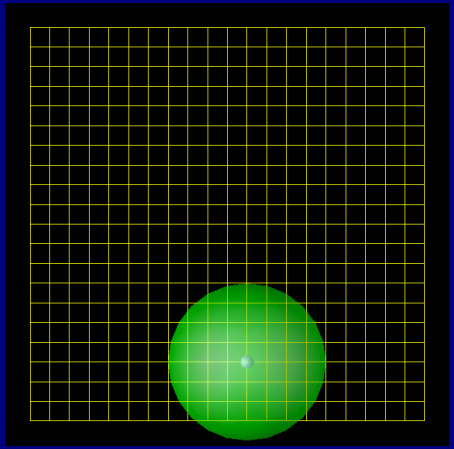


Tear also points in its vicinity *on the map*

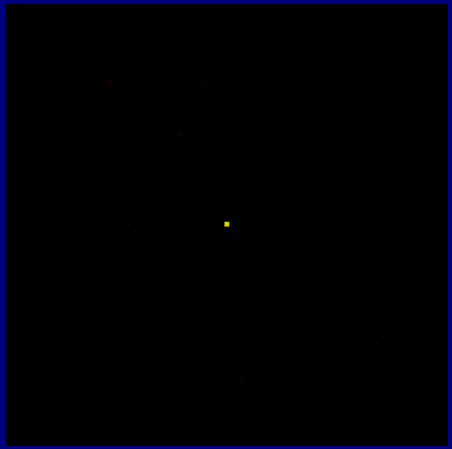


Tear it towards the data point

Iteration 1

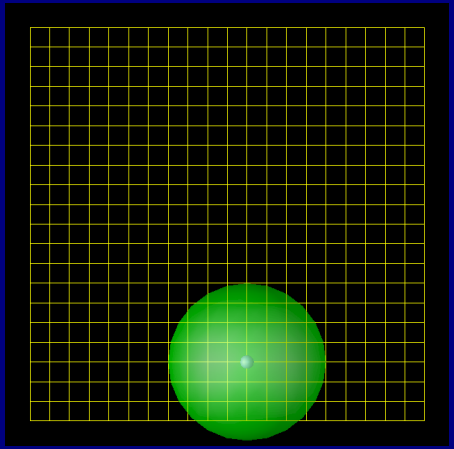


Tear also points in its vicinity *on the map*

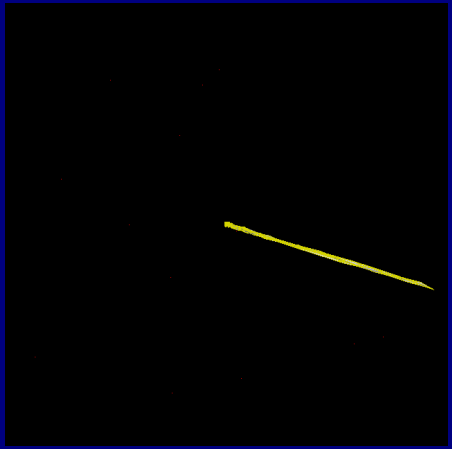


Zooming out. . .

Iteration 1

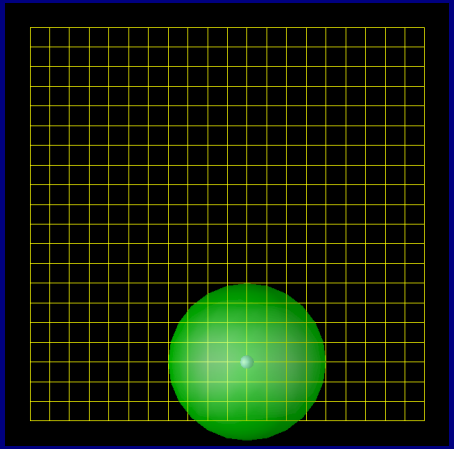


Tear also points in its vicinity *on the map*

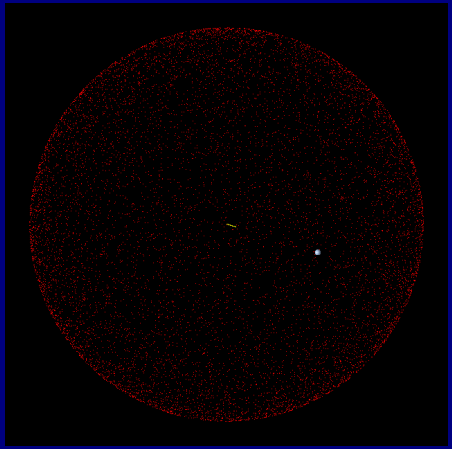


Torn!

Iteration 1

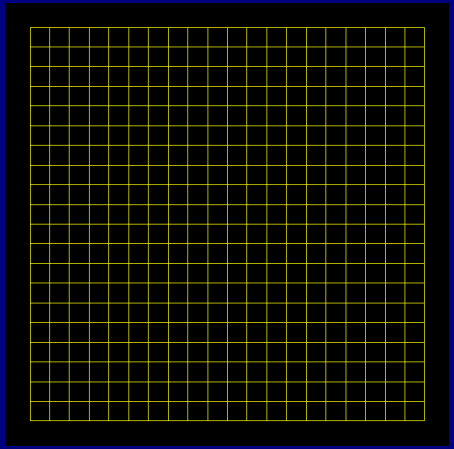


Tear also points in its vicinity *on the map*

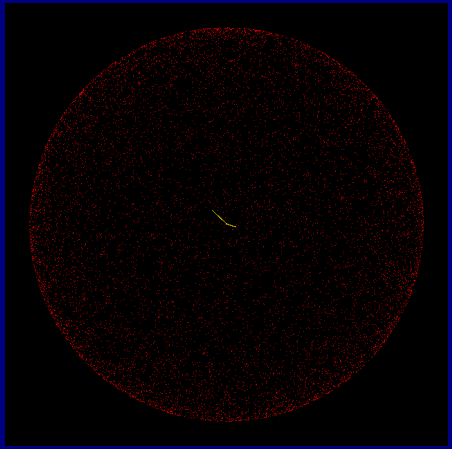


. . . towards the data point

Iteration 2

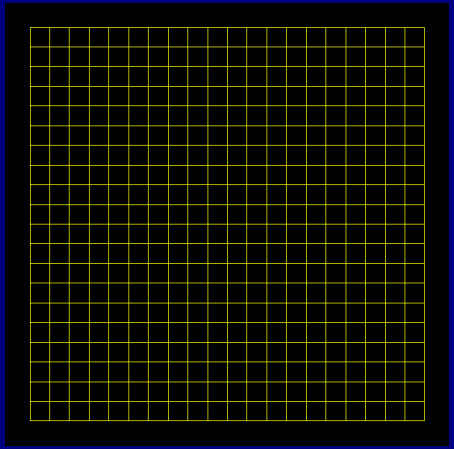


Low resolution (21×21) toy map

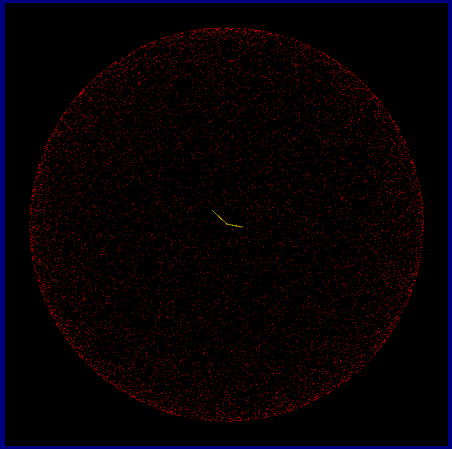


Probing the dataset...

Iteration 3

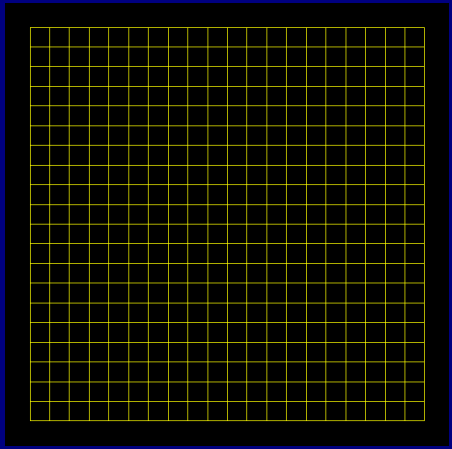


Low resolution (21×21) toy map

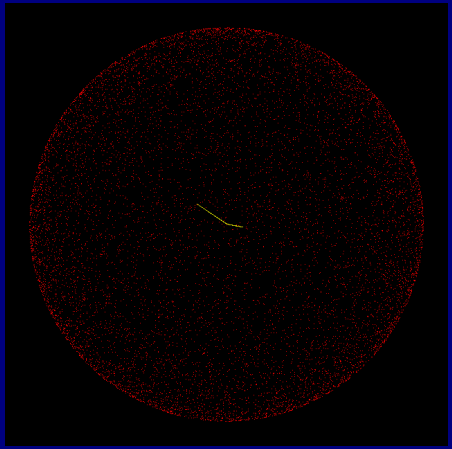


Probing the dataset...

Iteration 4

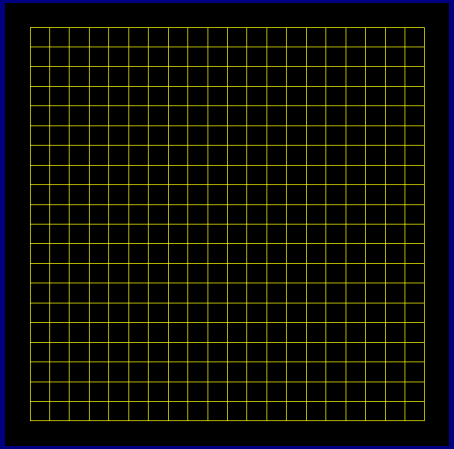


Low resolution (21×21) toy map

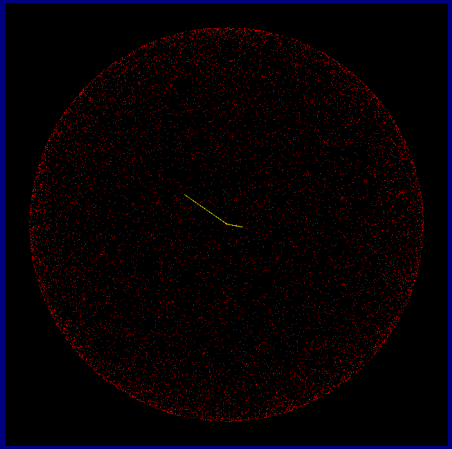


Probing the dataset...

Iteration 5

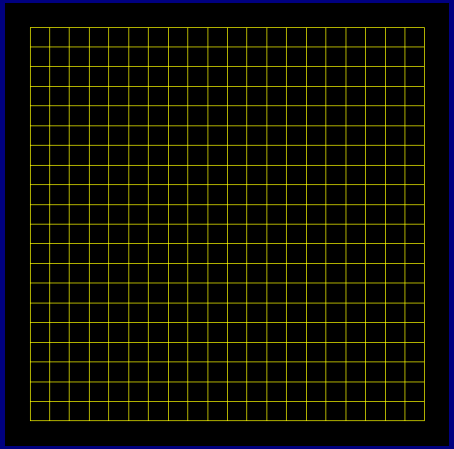


Low resolution (21×21) toy map

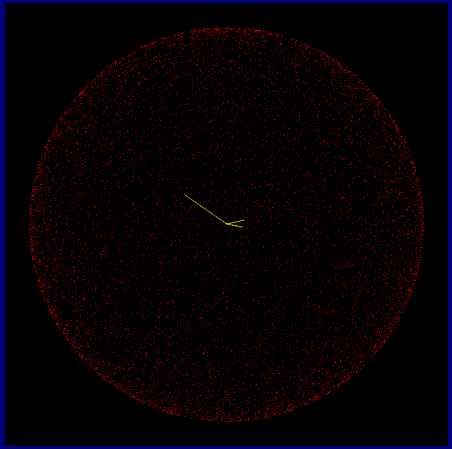


Probing the dataset...

Iteration 6

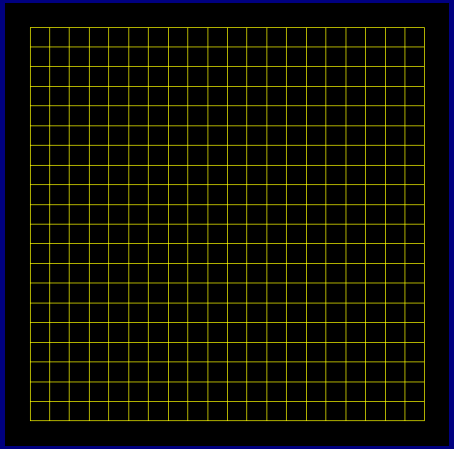


Low resolution (21×21) toy map

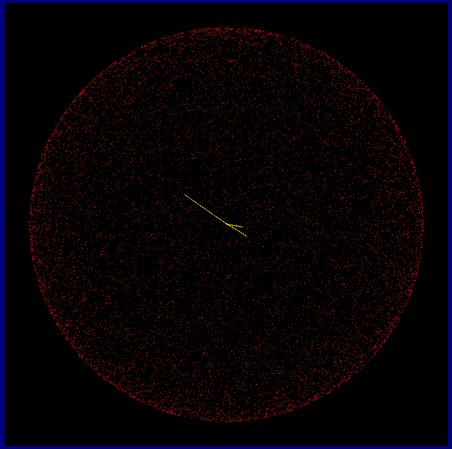


Probing the dataset...

Iteration 7

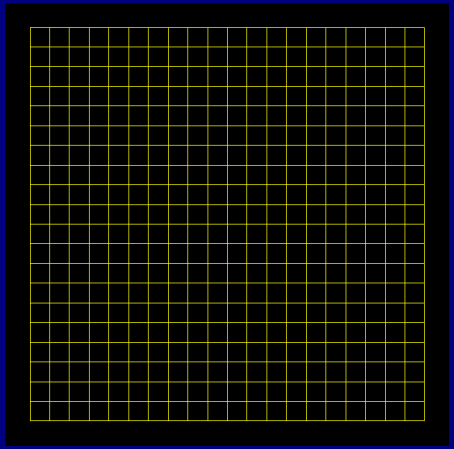


Low resolution (21×21) toy map

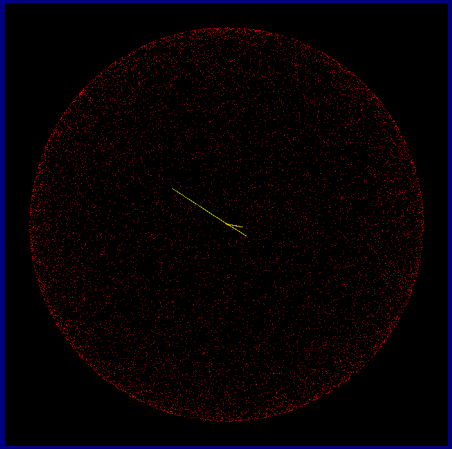


Probing the dataset...

Iteration 8

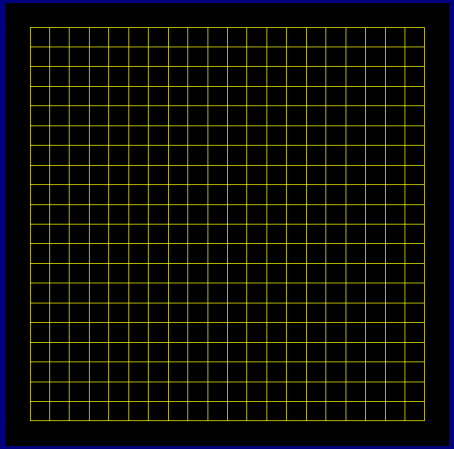


Low resolution (21×21) toy map

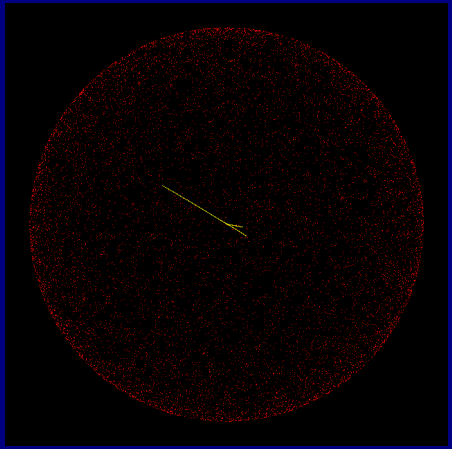


Probing the dataset...

Iteration 9

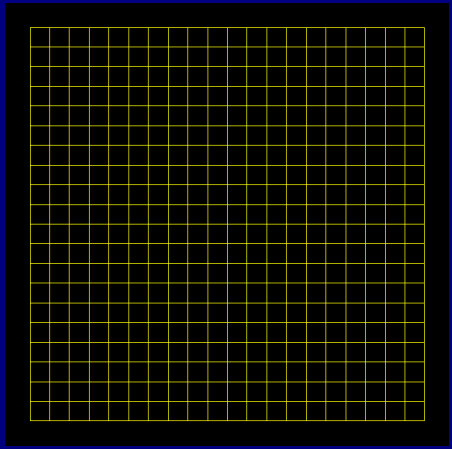


Low resolution (21×21) toy map

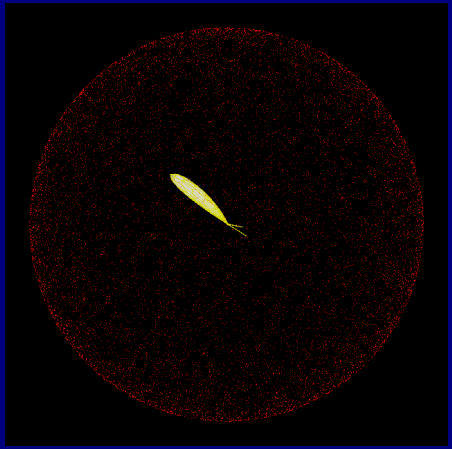


Probing the dataset...

Iteration 10

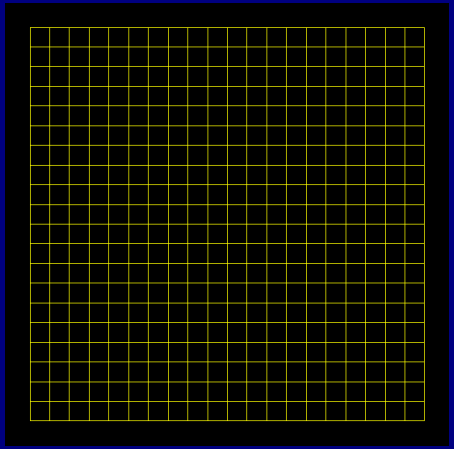


Low resolution (21×21) toy map

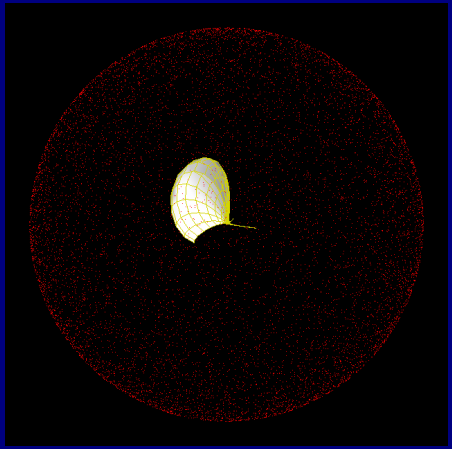


Probing the dataset...

Iteration 20

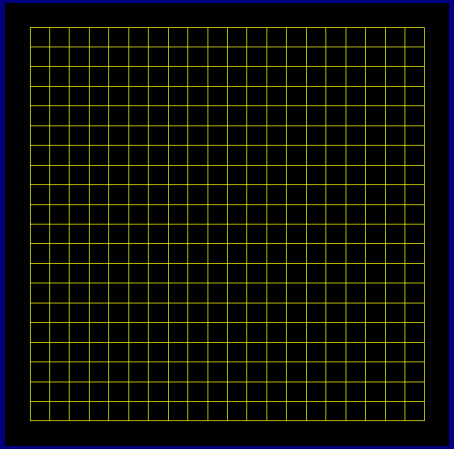


Low resolution (21×21) toy map

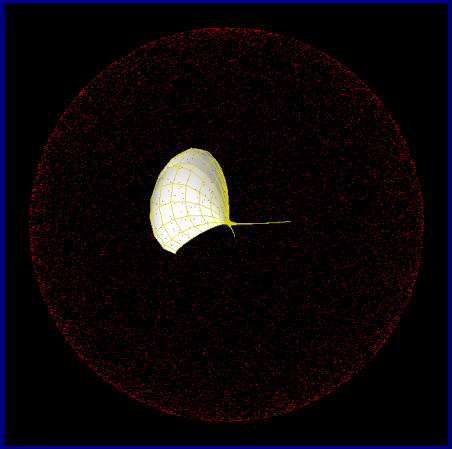


Unfolding...

Iteration 30

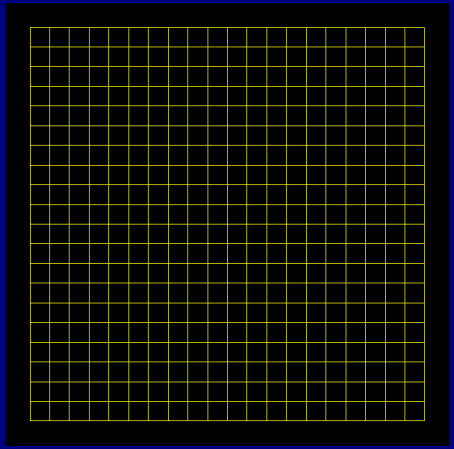


Low resolution (21×21) toy map

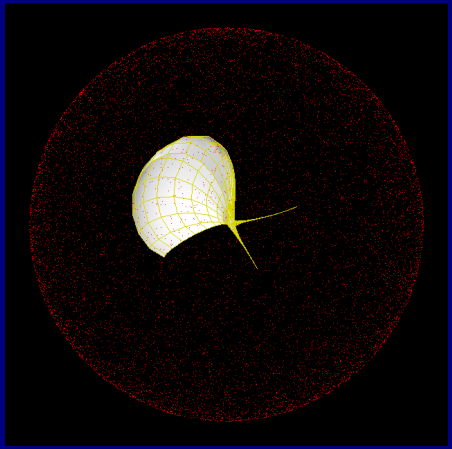


Unfolding...

Iteration 40

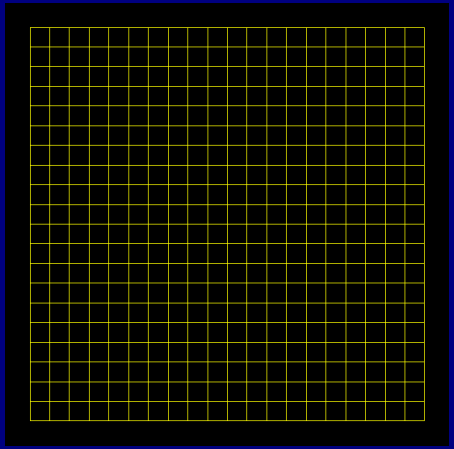


Low resolution (21×21) toy map

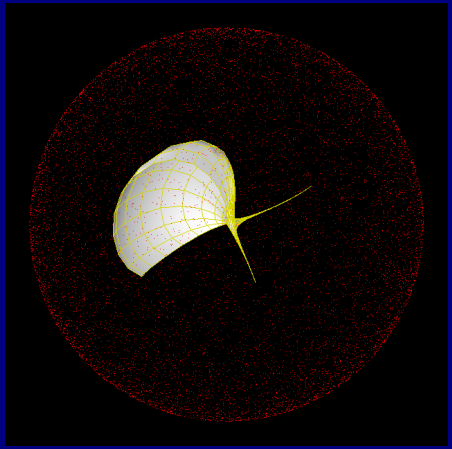


Unfolding...

Iteration 50

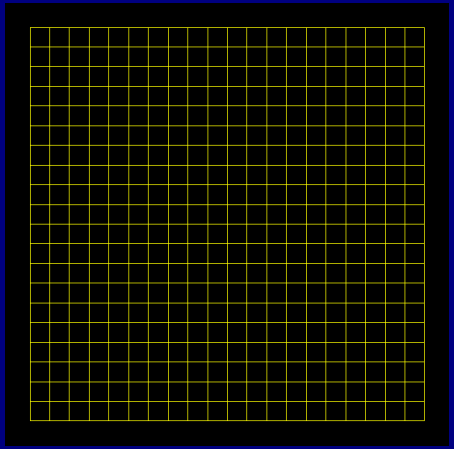


Low resolution (21×21) toy map

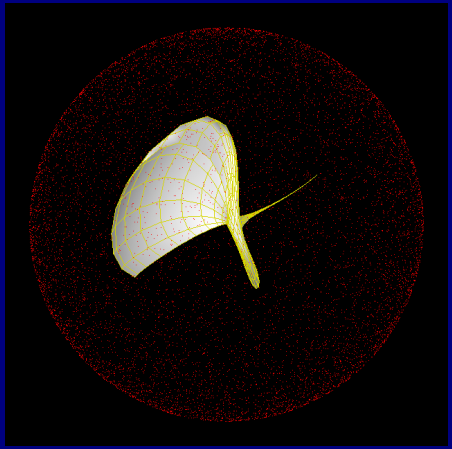


Unfolding...

Iteration 60

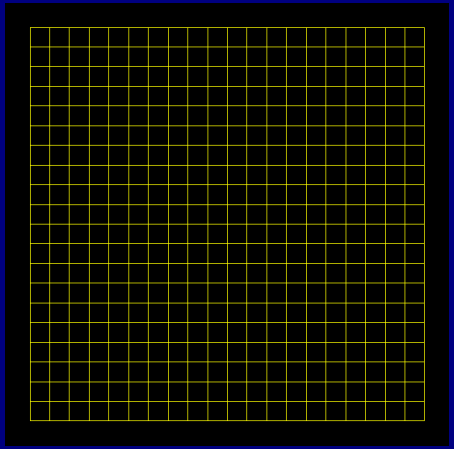


Low resolution (21×21) toy map

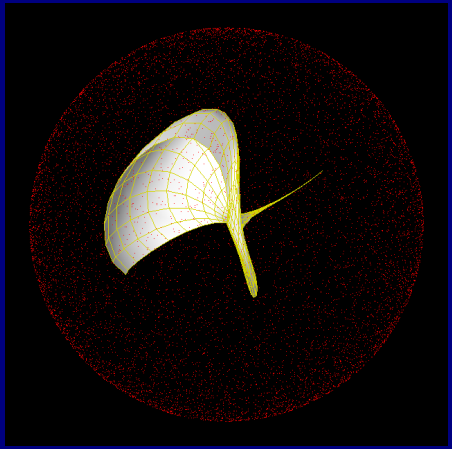


Unfolding...

Iteration 70

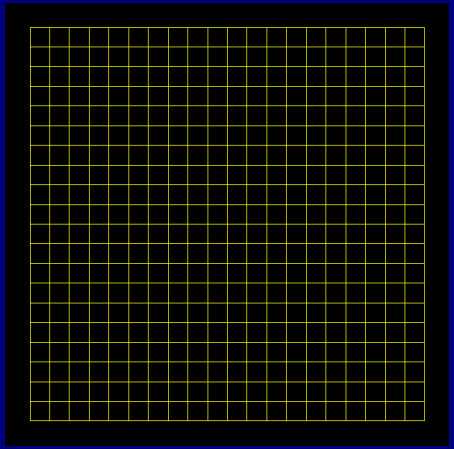


Low resolution (21×21) toy map

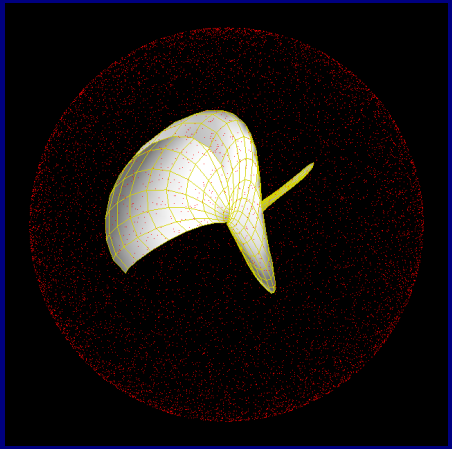


Unfolding...

Iteration 80

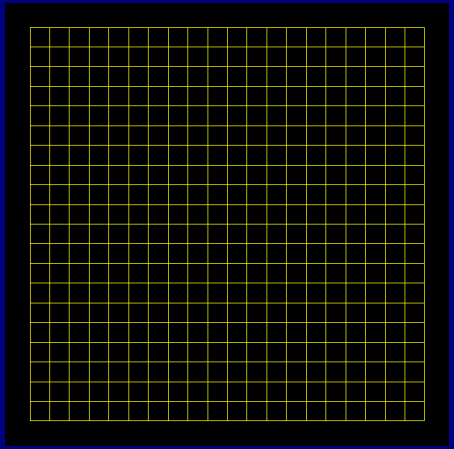


Low resolution (21×21) toy map

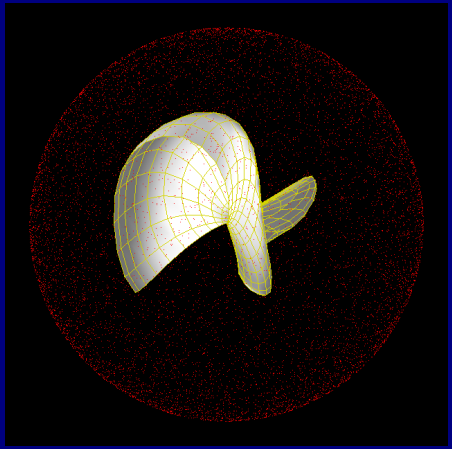


Unfolding...

Iteration 90

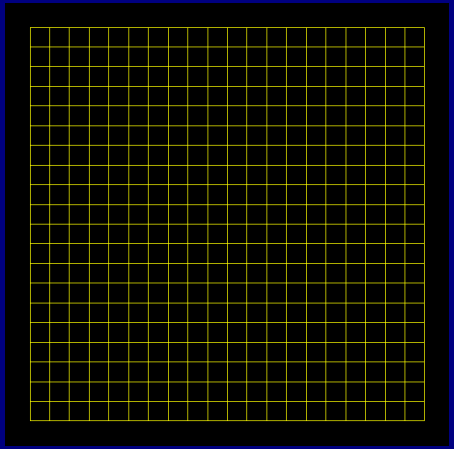


Low resolution (21×21) toy map

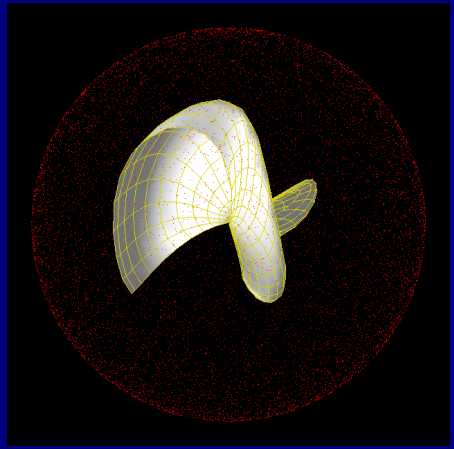


Unfolding...

Iteration 100

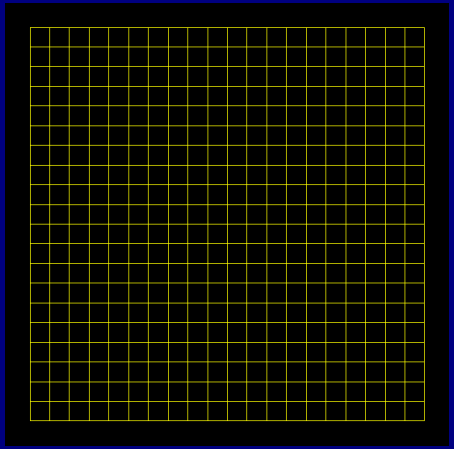


Low resolution (21×21) toy map

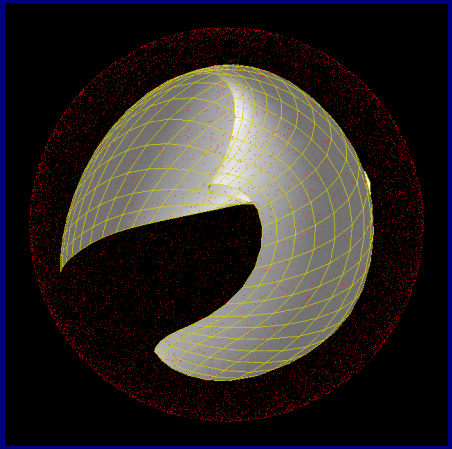


Unfolding...

Iteration 1000

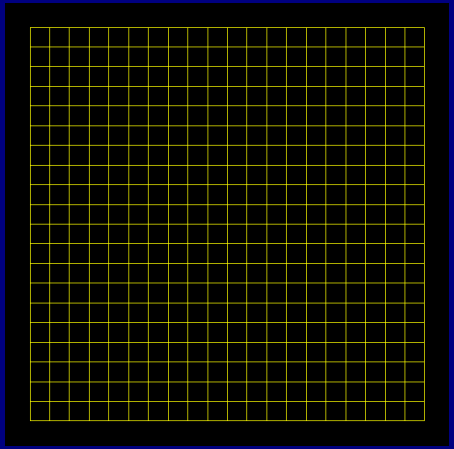


Low resolution (21×21) toy map

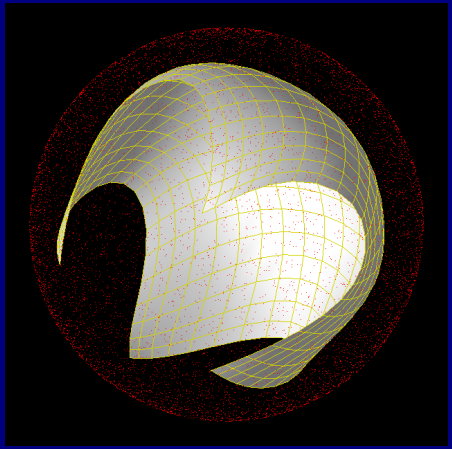


Unfolding...

Iteration 10000

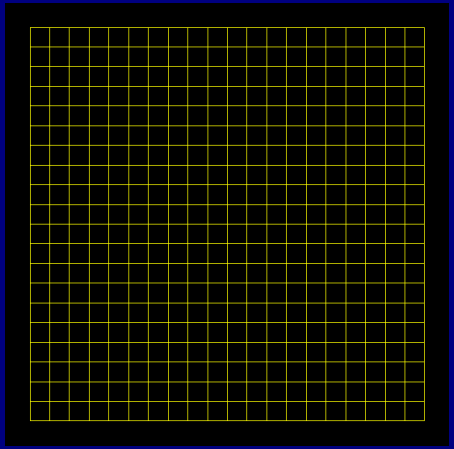


Low resolution (21×21) toy map

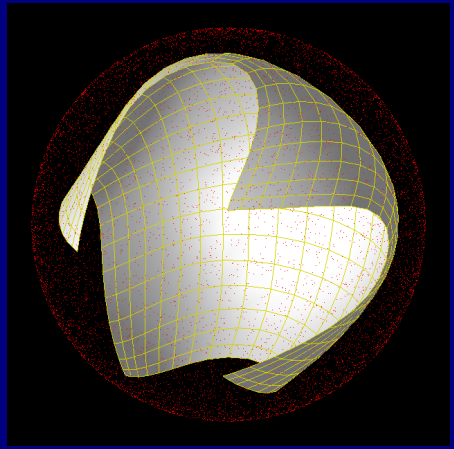


Adjusting...

Iteration 100000

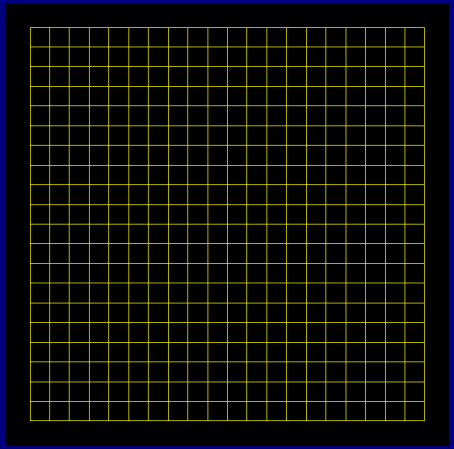


Low resolution (21×21) toy map

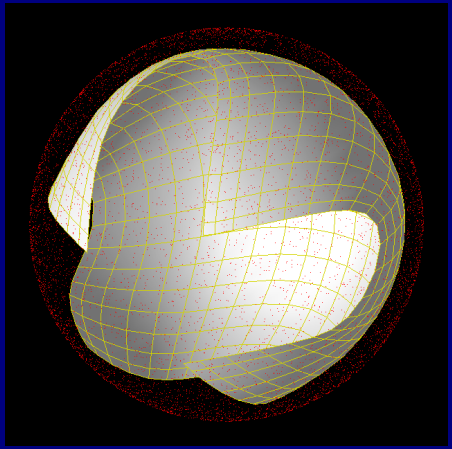


Adjusting...

Iteration 400000

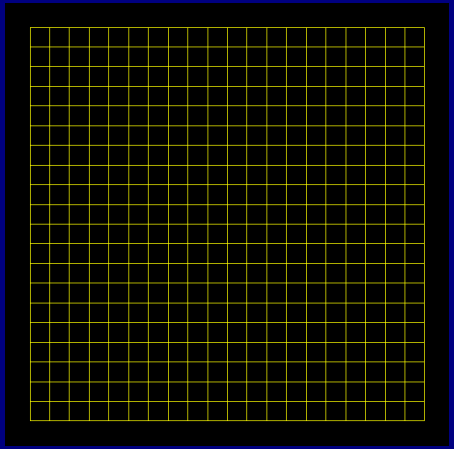


Low resolution (21×21) toy map

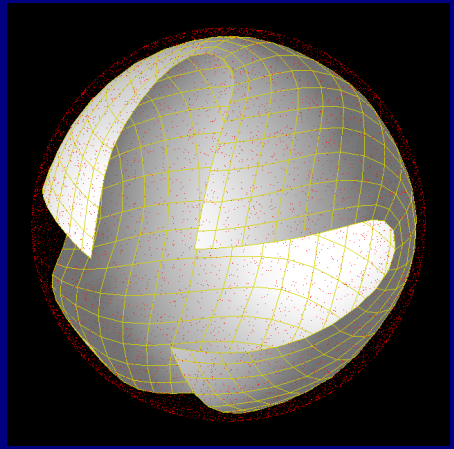


Adjusting...

Iteration 700000

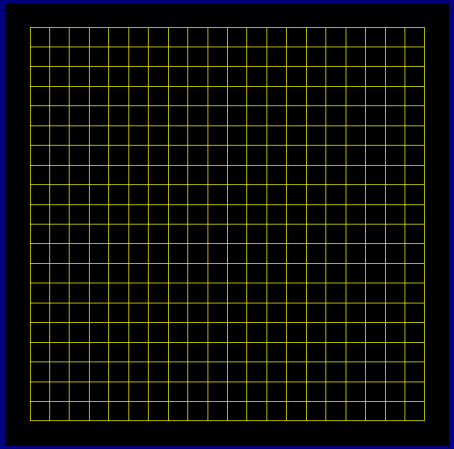


Low resolution (21×21) toy map

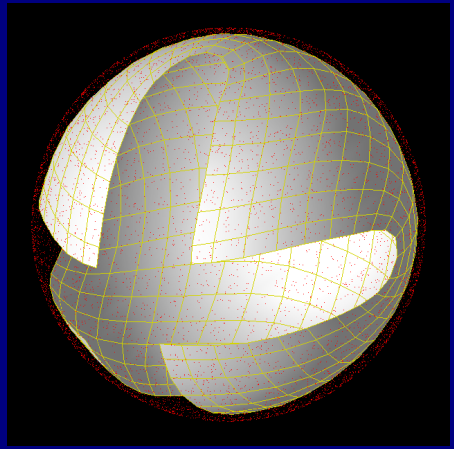


Adjusting...

Iteration 800000

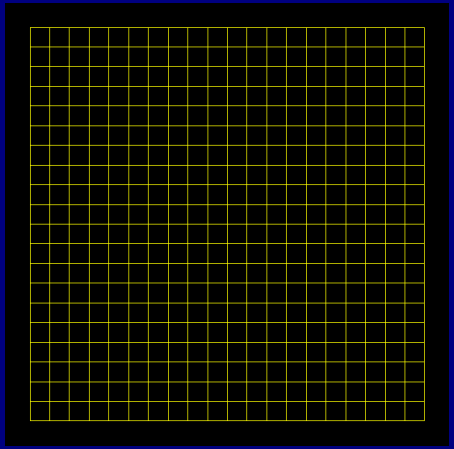


Low resolution (21×21) toy map

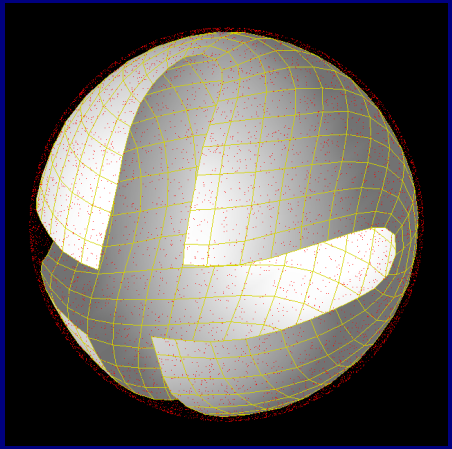


Adjusting...

Iteration 900000

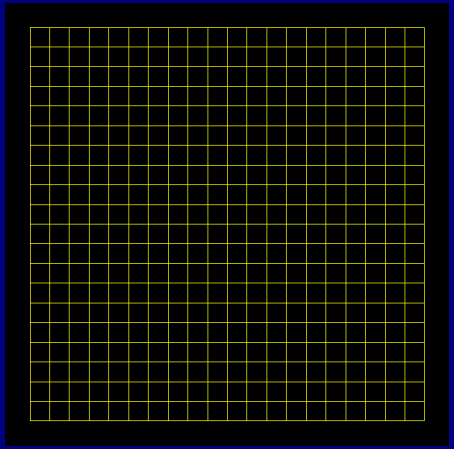


Low resolution (21×21) toy map

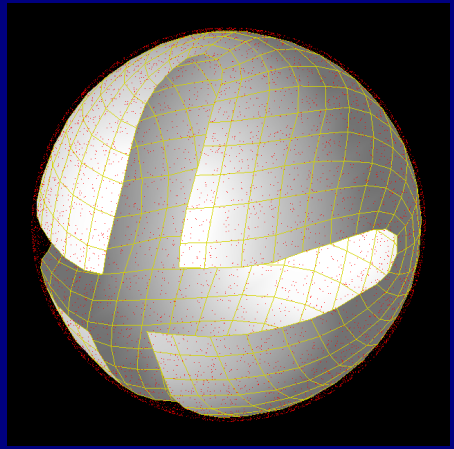


Adjusting...

Iteration 1000000



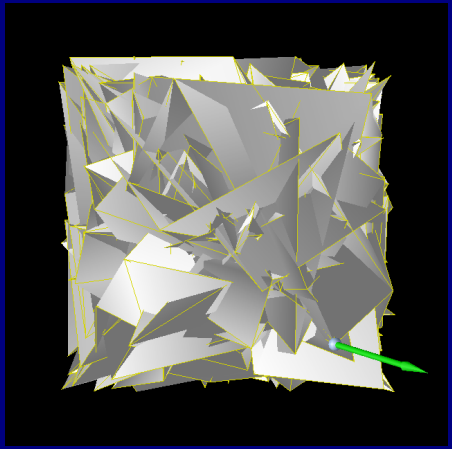
Low resolution (21×21) toy map



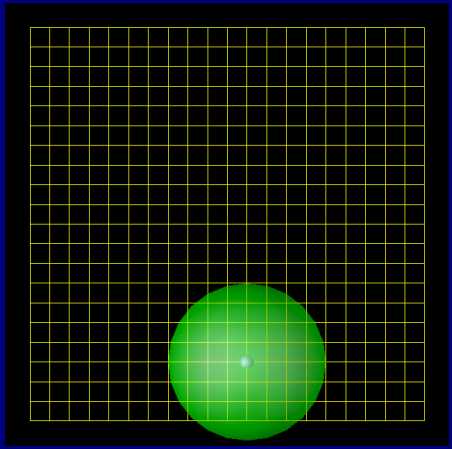
Done!

Learning parameters

- Distance by which the map grid point is torn towards the sample data point.
- It has to be turned down to zero towards the end of the optimization to avoid noise from the random presentation of sample points.



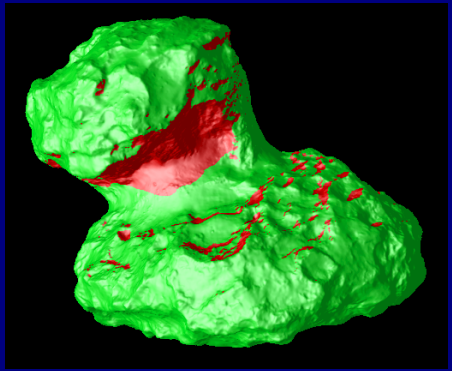
Learning parameters



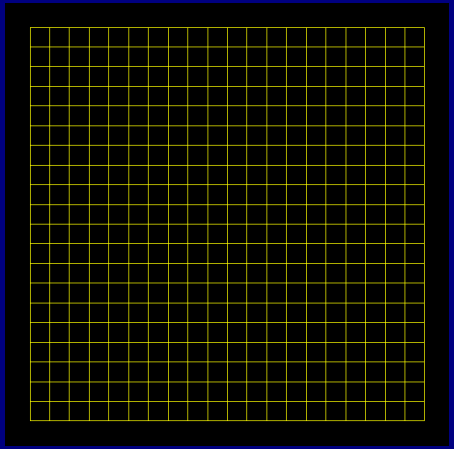
- Radius up to which points in the vicinity of the closest point are also torn towards the sample data point.
- It has also to be turned down to zero.
- It is needed in the beginning for the formation of a topologically correct map, but it would smooth the map unnecessarily in the end.

Comet 67P/Churyumov-Gerasimenko

- Common map projections cannot display the complete surface of Rosetta's target comet because of the overhung areas.
- There are multiple points with the same longitude and latitude.

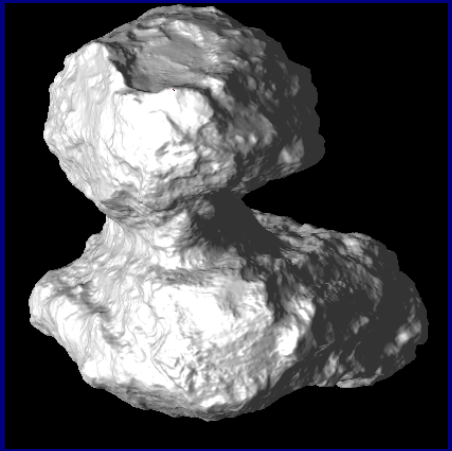


Comet 67P/Churyumov-Gerasimenko



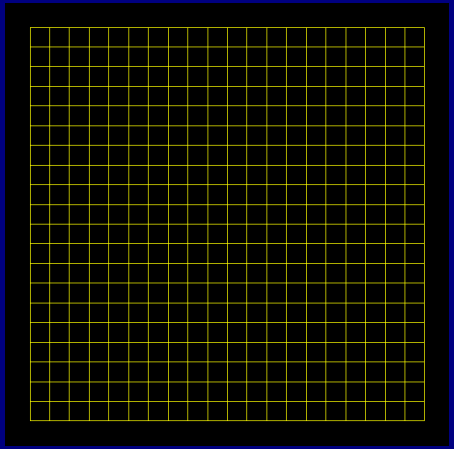
Low resolution (21×21) toy map

$\longrightarrow$

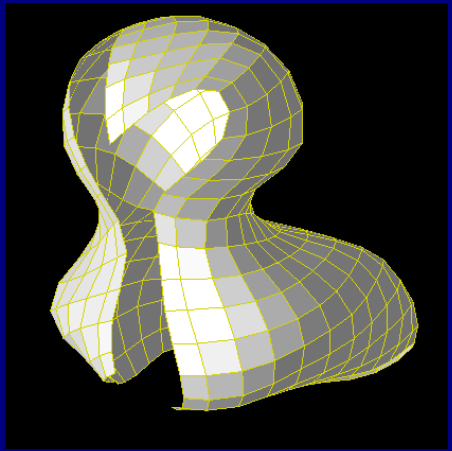


600 000 vertex points of the SHAP5 shape model

Comet 67P/Churyumov-Gerasimenko



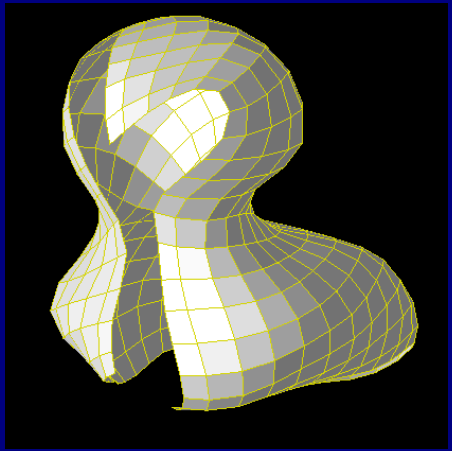
Low resolution (21×21) toy map



A coat for the duck — but without zipper.

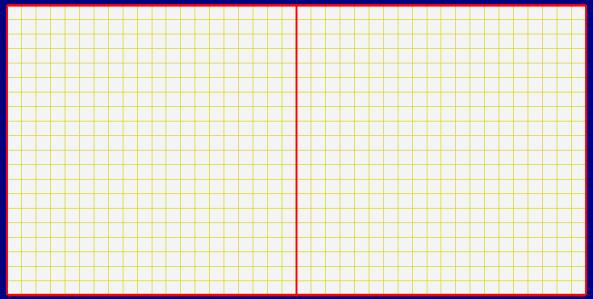
Comet 67P/Churyumov-Gerasimenko

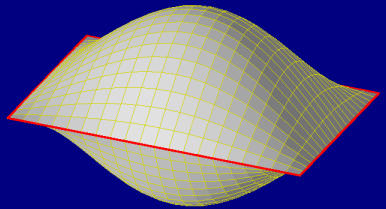
- The gap gets narrower with increasing grid resolution, but it never closes.
- To really cover the complete comet, we created the Quincuncial Adaptive Closed Kohonen (QuACK) map.
- Beyond closing the gap, this turned out to provide other far reaching benefits that we did not anticipate.

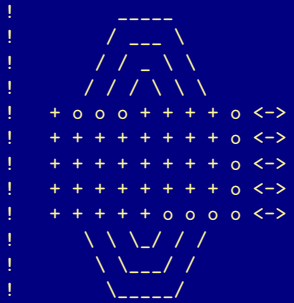


A coat for the duck — but without zipper.

Closing the map

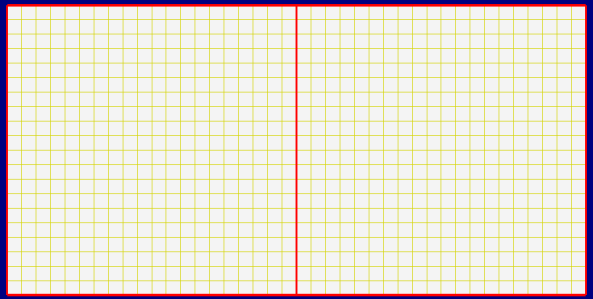


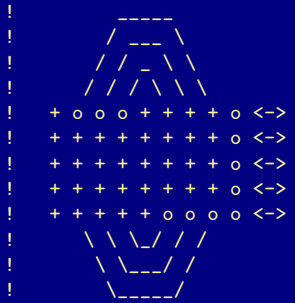


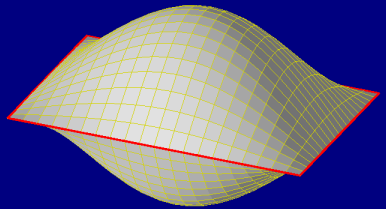


- We take two squares and sew them together at all four edges.
- I put the above in my code as cheat sheet.

Closing the map

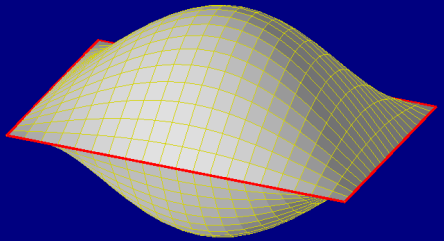


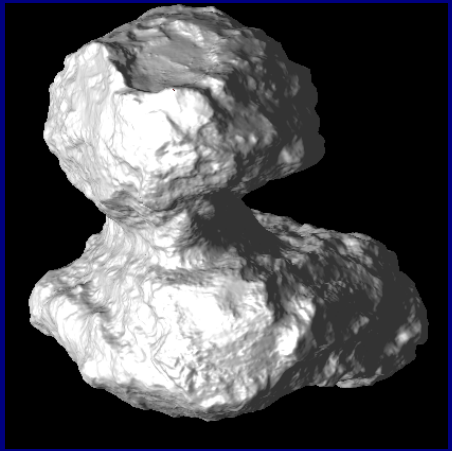




- \+ are vertices which are actually optimized.
- o are vertices which get a copy after the fact.
- We have to compute map distances across edges. 🤔

Fitting the QuACK map to Chury

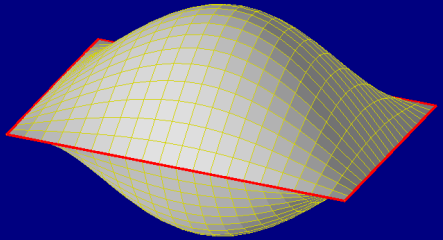


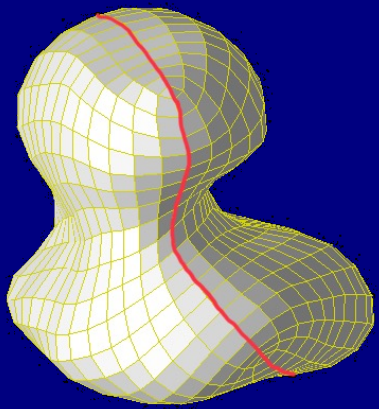


Closed low resolution (41×21) toy map

600 000 vertex points of the SHAP5 shape model

Fitting the QuACK map to Churyumov–Gerasimenko

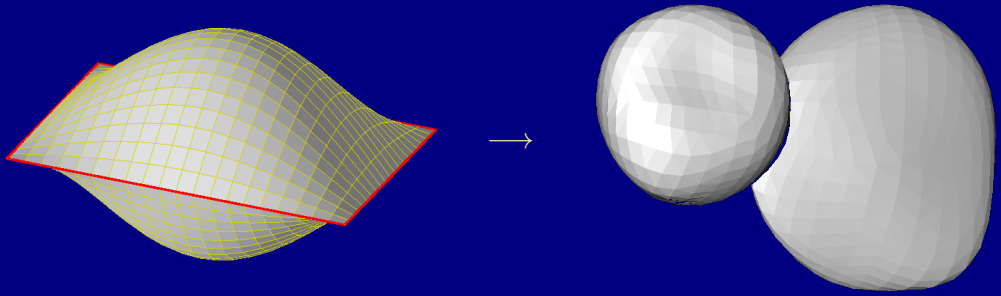




Closed low resolution (41×21) toy map

An unbroken space suit for the duck.

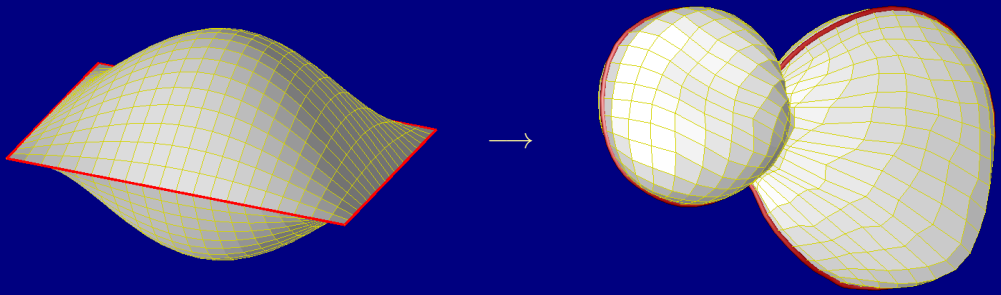
Fitting the QuACK map to Arrokoth



Closed low resolution (41×21) toy map

1046 vertex points of the Arrokoth shape model (2019)

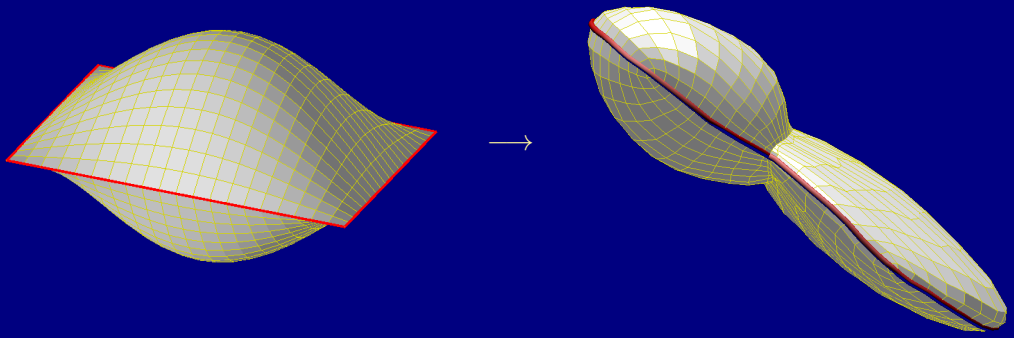
Fitting the QuACK map to Arrokoth



Closed low resolution (41×21) toy map

Unfoldable QuACK shape model

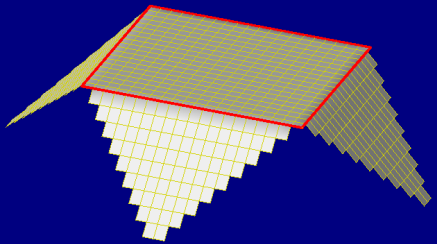
Fitting the QuACK map to Arrokoth



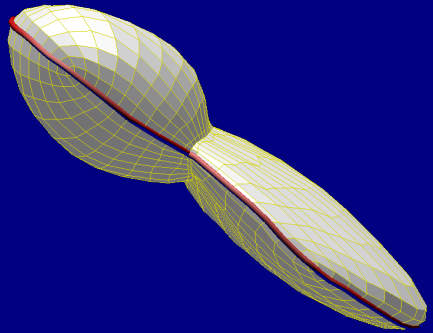
Closed low resolution (41×21) toy map

Unfoldable QuACK shape model

Unfolding the QuACK map

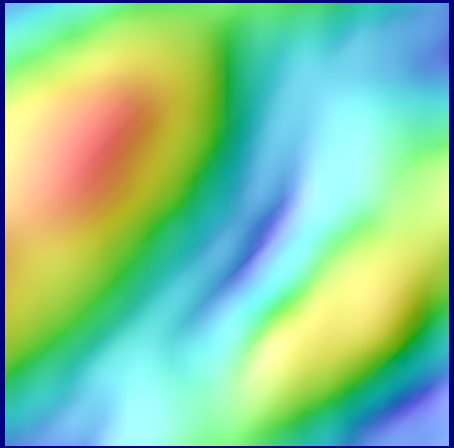


Unfolding to quincuncial layout



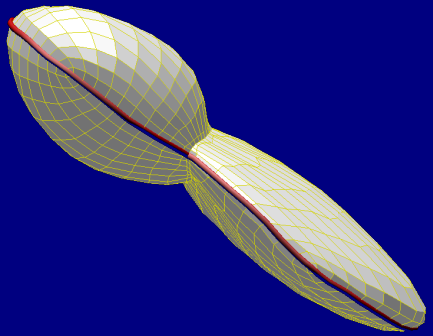
Unfoldable QuACK shape model

Unfolding the QuACK map



Color coded distance from the center in QuACK map projection

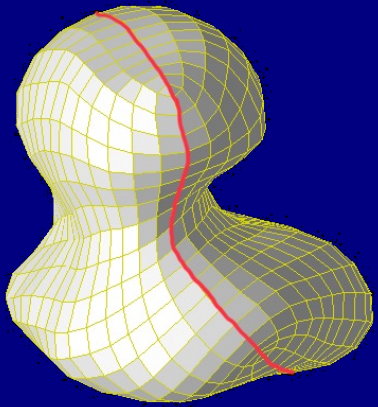
←



Unfoldable QuACK shape model

High resolution version for Chury

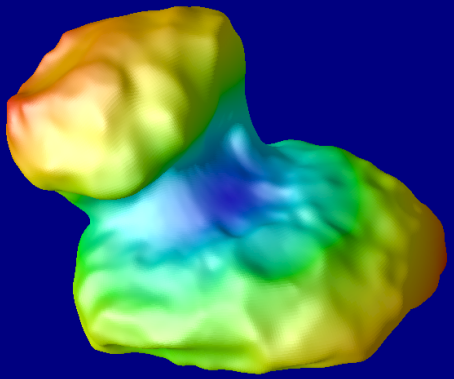
Toy model: 41 × 21 grid points



Toy map

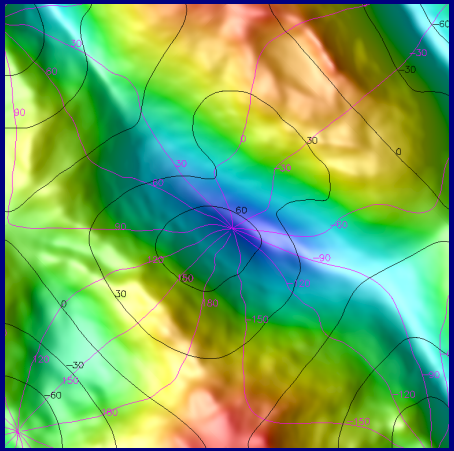
High resolution version for Chury

Toy model: 41 × 21 grid points
Full model: 401 × 201 grid points



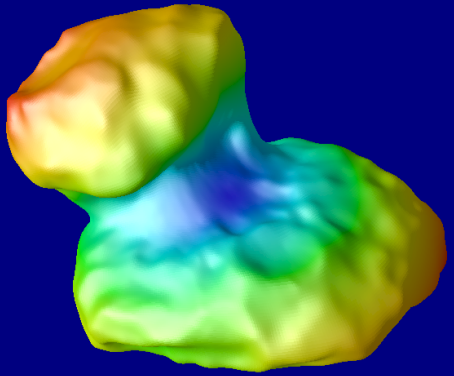
Full map
(Color encodes distance from center)

High resolution version for Chury



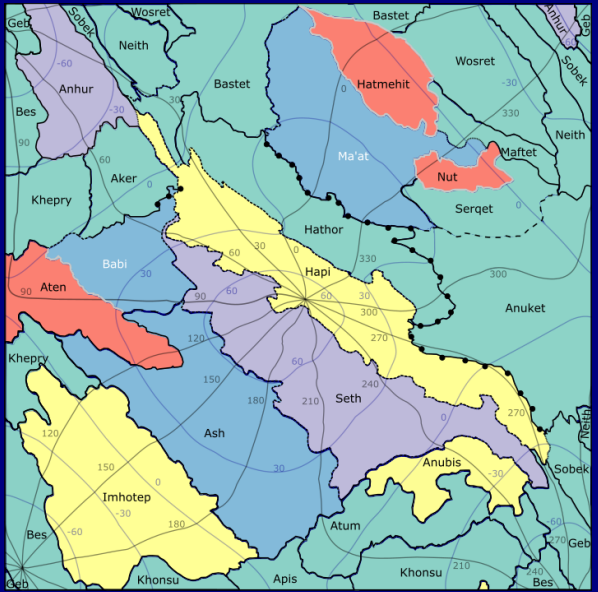
Unfolded full QuaCK map

←



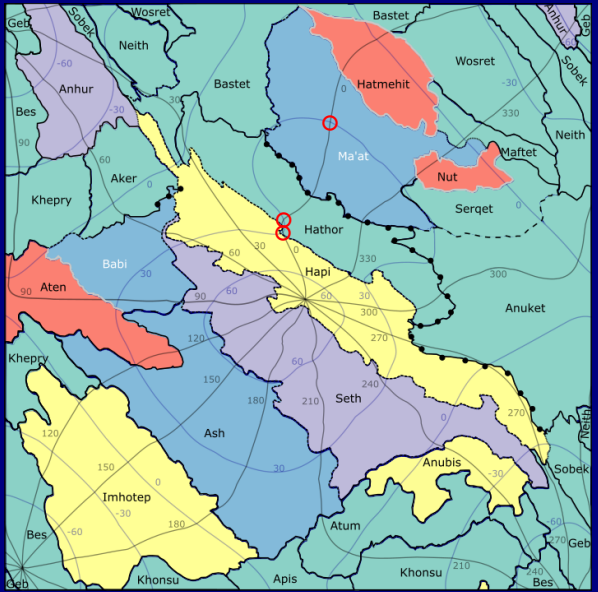
Adapted full QuACK map

Regions in the QuACK map projection



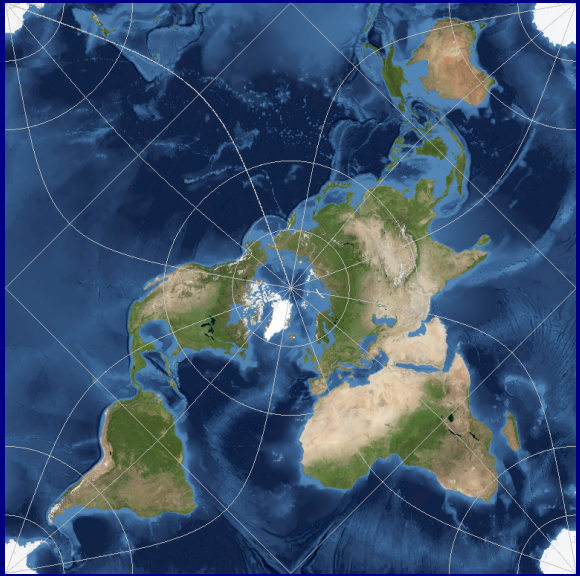
- Region boundaries (and any features) are mapped to the QuACK map with sub-grid accuracy by bilinear interpolation.

Regions in the QuACK map projection



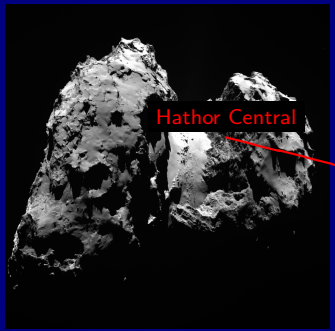
- Region boundaries (and any features) are mapped to the QuACK map with sub-grid accuracy by bilinear interpolation.
- Longitude and latitude lines provide an example of three different points with the same coordinates.

Peirce quincuncial projection of the world (1879)

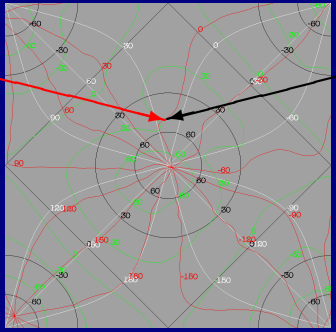


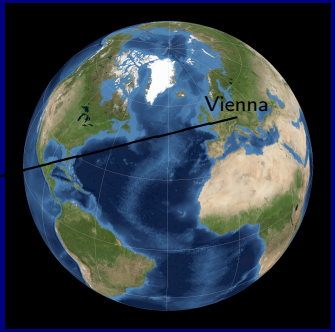
- The Northern hemisphere is mapped to the inner square (standing on a corner).
- The Southern hemisphere is cut into four triangles, with the South pole in all four corners of the outer square.
- This is topologically equivalent to the QuACK map.

Assigning generalized longitude and latitude



32°N, 0°E

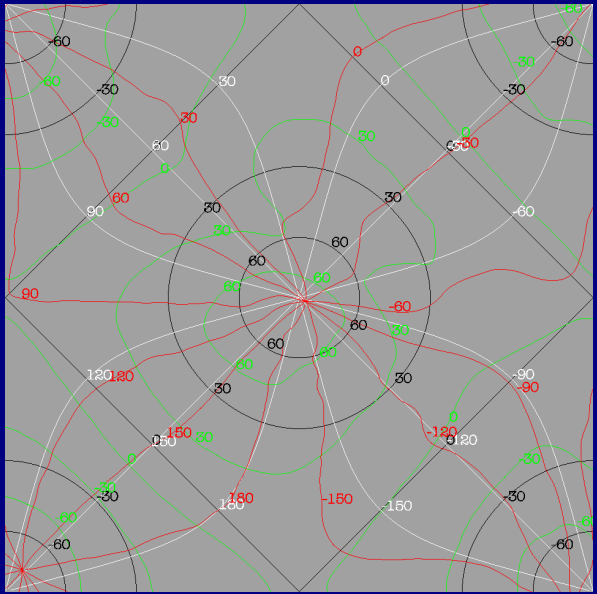




48°N, 16°E

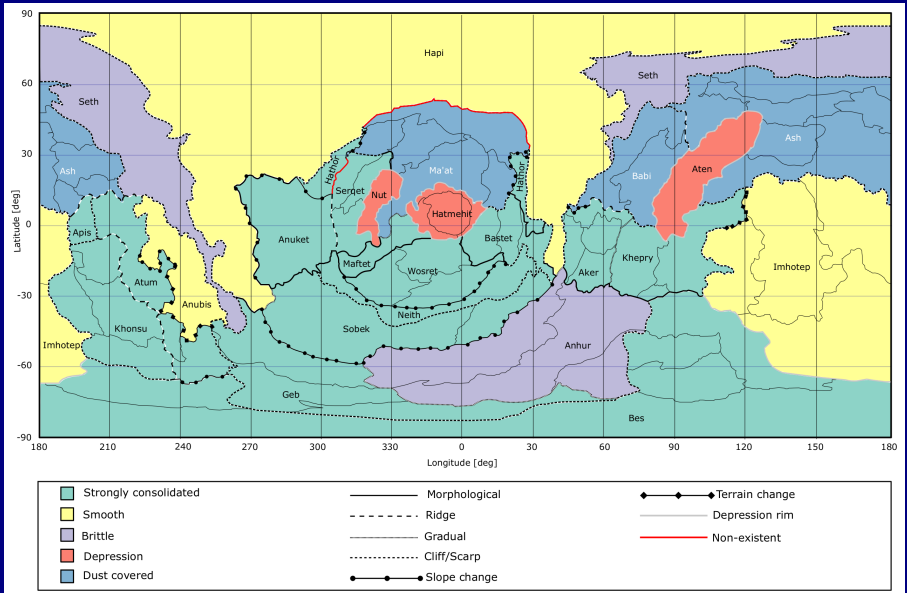
- Hathor Central and Vienna map to the same point on the QuACK map (respective Peirce quincuncial).
- We assign to Hathor Central the longitude and latitude of Vienna.
- Such **generalized** longitudes and latitudes are unambiguous over the comet.

Comparing actual and generalized longitude and latitude

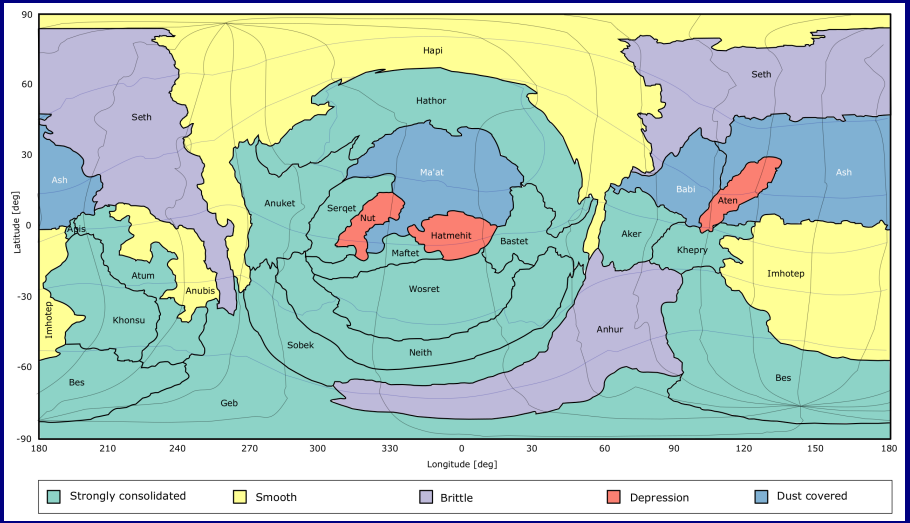


- The original comet longitudes and latitudes can be identical for different points.
- The assigned generalized longitudes and latitudes are unambigous.
- These can be used with any map projection — e. g., cylindrical equidistant — to obtain an unambigous generalized version.

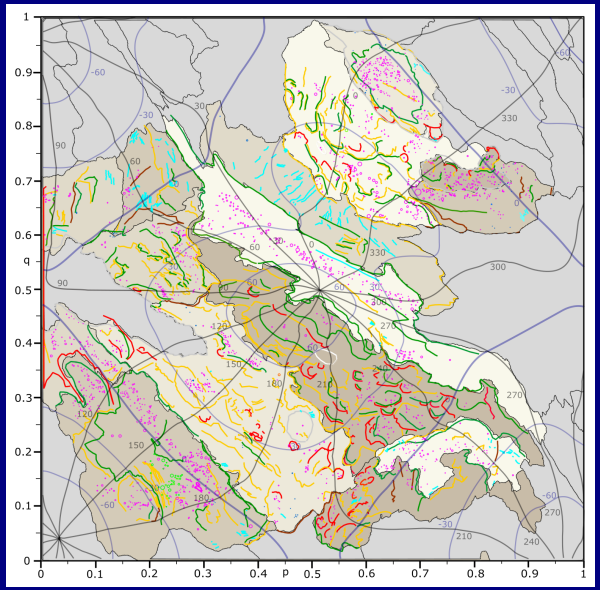
Common equidistant cylindrical projection

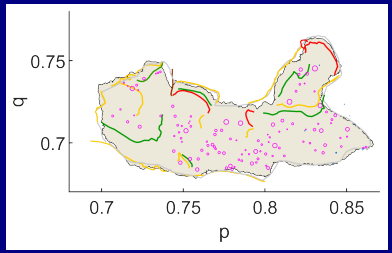


Generalized equidistant cylindrical projection

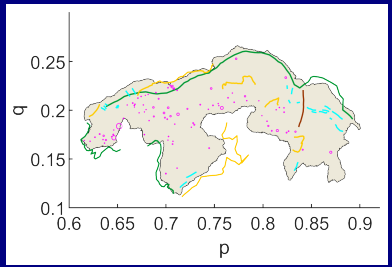


Geological maps





Nut



Anubis

Summary and outlook

- The QuACK map solves the problem of obtaining an unambiguous 2D projection for the complete surface of highly irregular 3D shapes.
- The essential extension of the regular Kohonen map is the construction of a closed topology by sewing together two squares at all four edges.
- A QuACK map has been (self-)adapted to comet 67P/Churyumov-Gerasimenko and applied for geological mapping.
- A QuACK map has also been (self-)adapted to the Kuiper belt object Arrokoth (though nothing has yet been mapped to it).
- A stereo derived higher resolution shape model of the sunward facing side of Arrokoth (Spender et al. 2020) is being merged with the global shape model. We will also fit a QuaCK map to the merged shape model.

References (1)

QuACK map projection

- Grieger, B. (2019). "Quincuncial adaptive closed Kohonen (QuACK) map for the irregularly shaped comet 67P/Churyumov-Gerasimenko". A&A 630, A1.
  https://doi.org/10.1051/0004-6361/201834841
- Software to apply the 67P QuACK map:
  https://github.com/esaSPICEservice/QuACK
- Kohonen, T. (1982). "Self-Organized Formation of Topologically Correct Feature Maps". Biological Cybernetics 43, 59–69.
  https://doi.org/10.1007/bf00337288

References (2)

Geological mapping of 67P

- Leon-Dasi, M., S. Besse, B. Grieger and M. Küppers (2021). “Mapping a Duck: Geological Features and Region Definitions on Comet 67P/Churyumov-Gerasimenko”. A&A 652, A52.
  https://doi.org/10.1051/0004-6361/202140497
- Products at ESAs Guest Storage Facility: European Space Agency, 2021, ESA-AURORA_67P-GEOMAP_OSIRIS_V1.0,
  https://doi.org/10.5270/esa-kokoti7
- Besse, S., M. Leon-Dasi, B. Grieger, and M Kueppers (2021). “Mapping a Duck: Geological Features and Region Definitions on Comet 67P/Churyumov-Gerasimenko”.
  https://doi.org/10.5194/epsc2021-43

References (3)

Quincuncial projection

- Peirce, C. S. (1879). “A quincuncial projection of the sphere”. American Journal of Mathematics 2 (4): 394–396. doi:10.2307/2369491. Available at https://www.jstor.org/stable/2369491

- Grieger, B. (2020). “Optimized global map projections for specific applications: the triptychial projection and the Spilhaus projection”. EGU2020-9885. https://doi.org/10.5194/egusphere-egu2020-9885